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ScienceDirect

Procedia Engineering 199 (2017) 1562–1567

Procedia Engineering

www.elsevier.com/locate/procedia

X International Conference on Structural Dynamics, EURODYN 2017

Numerical modeling of waveguides accounting for translational invariance and rotational symmetry

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Abstract

The analysis of high-frequency wave propagation in arbitrarily shaped waveguides requires specific numerical methods. A widely spread technique is the so-called semi-analytical finite element (SAFE) formulation. This formulation enables to account for the translational invariance of waveguide problems and leads to a two-dimensional modal problem reduced on the cross-section. Despite this, solving the problem can still be computationally demanding. In order to further reduce the size of the modal problem, this paper presents a SAFE method for waveguides of rotationally symmetric cross-sections. Such structures are encountered in many applications. Typical examples are bars of circular cross-section, regular polygons, and multiwire cables. Numerical results show that the computational effort required for solving the SAFE modal problem is tremendously reduced by accounting for rotational symmetry.

Peer-review under responsibility of the organizing committee of EURODYN 2017.

Keywords: waveguide ; finite element ; rotational symmetry ; modes ; bar ; cable

1. Introduction

Elastic guided waves are of great interest for the inspection of elongated structures. Guided waves are yet multimodal and dispersive, which complicates the physical interpretation of measurement. In practice, modeling tools are required to optimize inspection systems. The modeling of canonical geometries (plates, cylinders) can be done thanks to analytical methods (see *e.g.* [1]).

The analysis of arbitrarily shaped waveguides yet requires numerical methods. A widely spread technique is the so-called semi-analytical finite element (SAFE) formulation [2,3]. It consists in accounting for translational invariance by applying a Fourier transform in the axial direction before finite element discretization. This leads to a two-dimensional modal problem reduced on the cross-section. Despite this, solving the problem can still be computationally demanding when the FE mesh has to be refined and/or the modal density increases (e.g. at high frequency) [4,5]. It is hence desirable to further reduce the size of the modal problem.

This paper presents a SAFE method for rotational symmetric cross-sections, often encountered in practical applications (regular polygons for instance). With a modal approach, accounting for rotational symmetry has indeed two

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1877-7058
Peer-review under responsibility of the organizing committee of EURODYN 2017.
10.1016/j.proeng.2017.09.462

computational benefits: the reduction of the problem size in terms of degrees of freedom and the reduction of the number of modes to compute.

Of particular interest in this work is the modeling of cables in view of nondestructive evaluation. Cables are usually made of individual helical wires, which are coupled through contact conditions. The proper SAFE formulation must be written in a nontrivial helical coordinate system allowing translational invariance along the axis [6]. Such structures yield large size problems involving mesh refinements in the contact regions [5].

Section 2 briefly recalls the SAFE method. The rotational symmetric formulation is presented in Sec. 3. Section 4 is devoted to numerical results. The first test case will be a cylindrical waveguide to validate the approach. Then, a seven-wire strand will be considered. Seven-wire strands are constituted by one central cylindrical wire surrounded by six peripheral helical wires and are widely employed in civil engineering cables.

2. Background: SAFE formulation

The SAFE method is a finite element method dedicated to waveguides. This method aims to account for the translational invariance of the geometry to reduce the size of the problem. The initial full 3D problem is reduced to a 2D modal problem so that one only needs to mesh the cross-section of the waveguide. This section recalls the SAFE formulation. Details can be found in the literature (see *e.g.* [2,3]). Let us denote (x, y) the cross-section coordinates and z the axis coordinate of the waveguide.

First, the strain-displacement relation is written as:

$$\boldsymbol{\epsilon} = (\mathbf{L}_{xy} + \mathbf{L}_z \partial/\partial z)\mathbf{u} \tag{1}$$

where $\mathbf{L}_{xy}$ is the operator containing all terms but derivatives with respect to the z-axis and $\mathbf{L}_z$ is the operator of z-derivatives:

$$\mathbf{L}_{xy} = \begin{bmatrix} \partial/\partial x & 0 & 0 \\ 0 & \partial/\partial y & 0 \\ 0 & 0 & 0 \\ \partial/\partial y & \partial/\partial x & 0 \\ 0 & 0 & \partial/\partial x \\ 0 & 0 & \partial/\partial y \end{bmatrix}, \quad \mathbf{L}_z = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}. \tag{2}$$

Then, the SAFE approach consists in applying a time Fourier transform as well as a spatial Fourier transform along z before discretizing the cross-section (x, y) by a finite element method. Inside one finite element e, the displacement field can thus be expressed as follows:

$$\mathbf{u}(x, y, z, t) = \mathbf{N}^e(x, y)\mathbf{U}^e e^{i(kz-\omega t)} \tag{3}$$

where $\mathbf{U}^e$ is the nodal displacement vector and $\mathbf{N}^e$ is the matrix of nodal interpolating functions of the element e. k is the axial wavenumber and ω is the angular frequency.

The variational formulation of three-dimensional elastodynamics yields, from Eqs. (1)–(3), the following matrix equation:

$$\{\mathbf{K}_1 - \omega^2\mathbf{M} + ik(\mathbf{K}_2 - \mathbf{K}_2^{\mathrm{T}}) + k^2\mathbf{K}_3\}\mathbf{U} = \mathbf{F} \tag{4}$$

with the elementary matrices:

$$\begin{aligned} \mathbf{K}_1^e &= \int_{S^e} \mathbf{N}^{e\mathrm{T}}\mathbf{L}_{xy}^{\mathrm{T}}\mathbf{C}\mathbf{L}_{xy}\mathbf{N}^e \mathrm{d}S, \quad \mathbf{K}_2^e = \int_{S^e} \mathbf{N}^{e\mathrm{T}}\mathbf{L}_{xy}^{\mathrm{T}}\mathbf{C}\mathbf{L}_z\mathbf{N}^e \mathrm{d}S, \\ \mathbf{K}_3^e &= \int_{S^e} \mathbf{N}^{e\mathrm{T}}\mathbf{L}_z^{\mathrm{T}}\mathbf{C}\mathbf{L}_z\mathbf{N}^e \mathrm{d}S, \quad \mathbf{M}^e = \int_{S^e} \rho\mathbf{N}^{e\mathrm{T}}\mathbf{N}^e \mathrm{d}S \end{aligned} \tag{5}$$

where $\mathrm{d}S = \mathrm{d}x\mathrm{d}y$ and $\mathbf{C}$ is the matrix of material properties. Setting $\mathbf{F} = \mathbf{0}$ (no acoustic source), Eq. (4) is an eigenvalue problem whose eigensolutions are the guided modes propagating in the translationally invariant structure.

To enforce rotational symmetry, periodic boundary conditions will be needed both on $\mathbf{U}$ and $\mathbf{F}$ as explained in the next section.