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A first-order numerical approach to switched-mode systems optimization

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ABSTRACT

This paper studies optimal control processes governed by switched-mode systems. We consider Optimal Control Problems (OCPs) with smooth cost functionals and apply a newly elaborated abstraction for the system dynamics under consideration. The control design we finally obtain includes an optimal switching times selection ("timing") as well as an optimal modes sequence scheduling ("sequencing"). For purpose of numerical treatment of the initially given OCP we use a newly elaborated relaxation concept and analyse the resulting "weakly relaxed" optimization problems. In contrast to the conventional relaxations our approach is based on the infimal prox convolution technique and does not use the celebrated Chattering Lemma. This fact causes a lower relaxation gap. Our aim is to propose a gradient-based computational algorithms for the OCPs with switched-mode dynamics. In particular, we deal with the celebrated Armijo-type gradient methods and establish the corresponding convergence properties. The numerical consistency (numerical stability) analysis makes it possible to apply a class of relative simple first-order numerical procedures to a sophisticated initial OCP involved in specific switched-mode dynamics.

1. Introduction

Optimization based modelling approach associated with sophisticated dynamic models and the corresponding numerical algorithms are nowadays a usual and relative mature design methodology for the software oriented development of several types of modern controllers (see e.g., [1–24]). Our paper considers a particular class of switched dynamics, namely, so called switched-mode control systems and focuses on the computational methods for OCPs involving relaxations procedures. Using the fundamental concepts and definitions from [25–29] we consider here so called "autonomous" switched-mode dynamic models. Recall that the autonomous and controlled switched-mode control systems and the related OCPs have been comprehensively studied in the past several years due to their engineering applications. Let us mention here some modern real-world applications from the mobile robot technology, automotive control, telecommunications and process control. An interesting application of the optimal switched-mode dynamics to the control systems powered by a specific multiple data sources is discussed in [17]. We refer to [1–3,25,10,11,30,14,19] and to [26–29] for various examples of OCPs with switched-type dynamics and for engineering implementations of the optimized switched-mode systems.

The main aim of our contribution is to elaborate a consistent and at the same time relative simple computational algorithm for OCPs associated with autonomous switched-mode systems. This optimization approach includes an optimal

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sequencing scheduling as well as a simultaneous optimal timing (see [27] for concepts). Let us note that the previously obtained numerical schemes and results for example, the so called "mode-insertion algorithm" and similar [25,11,30] usually include two separate optimization steps with respect to the timing/sequencing. The widely used "gradient-descent" methodology (see [10,15,11,26–29]) is not sufficiently extended by the necessary numerical consistency investigation. The convergence proof for this method is not fully completed. In our paper, we propose an effective optimization method based on a specific combination of the novel relaxation schemes for OCPs and the gradient-projection approach. The new relaxation scheme we develop involves a lower "relaxation gap" effect in comparison to the classic generalization schemes based on the well-known Chattering Lemma (see Section 6). Moreover, we also propose a new conceptual relaxation approach to a specific class of problems, namely, to singular OCPs involving switched-mode systems (see Section 5). And, it should be noted already at this point that a computational scheme we propose can be effectively used in a concrete switched-mode systems optimization phase and moreover, it possesses the strong numerical consistency properties. The corresponding convergence result (Theorem 2) constitutes the main theoretic part of our contribution. Note that this convergence result (for the concrete class of OCPs under consideration) is stricter than the corresponding result based on the classic Chattering Lemma (see [5,7,9,31,21,22]). This fact is an immediate consequence of the lower relaxation gap mentioned above. Let us also emphasize that the application of the conventional relaxation schemes (see e.g., [32] and the references therein) to the general switched systems studied in [5] does not incorporate a specific class of systems with a possible sequence scheduling ("sequencing"). The resulting algorithm developed in our paper is finally applied to an illustrative computational example of OCP, namely, to the two tanks model optimization.

The remainder of this paper is organized as follows: Section 2 deals with a rigorous mathematical problem formulation. We give here a self-closed formal concept of the autonomous switched-mode control systems. Section 3 presents a fully relaxed (convexified) scheme associated with the initially given non-convex OCP. This section also includes a projected gradient approach to the simply relaxed OCPs. In Section 4 we study a newly developed infimal prox convolution technique for the given OCPs. Application of the convex infimal convolution idea to the switched-mode driven OCP makes it possible to obtain a weakly relaxed dynamic optimization problem. The last one provides a theoretical basis for the concrete numerical treatment of the initial OCPs. In Section 5 we consider the infimal based relaxation in the context of a singular OCP associated with the switched-mode dynamics. Section 6 discusses numerical aspects of the developed theory and also includes application of the proposed computational method to a two tanks model. Section 7 summarizes our paper.

2. Problem formulation and basic model

Consider a switched-mode control system dynamics in the form

$$\begin{aligned}&\dot{x}(t) = \sum_{i=1}^{I} \beta_{[t_{i-1},t_i)}(t) \sum_{k=1}^{K} q_{k,[t_{i-1},t_i)}(t) f_k(x(t))\\ &\text{a.e. on } [0, t_f], \quad x(0) = x_0 \in \mathbb{R}^n.\end{aligned} \tag{1}$$

Here $x(t) \in \mathbb{R}^n$ denotes a state vector, $f_k : \mathbb{R}^n \to \mathbb{R}^n$ for $k = 1, \ldots, K \in \mathbb{N}$ are continuously differentiable function. By

$$\beta_{[t_{i-1},t_i)}(t) = \begin{cases} 1 & \text{if } t \in [t_{i-1}, t_i) \\ 0 & \text{otherwise} \end{cases}$$

we denote the classic characteristic functions associated with the disjunct time intervals $[t_{i-1}, t_i)$. Let $t_0 = 0$, $t_I = t_f$. We assume that I is a finite (non-fixed a priori) natural number and $q_{k,[t_{i-1},t_i)}(t) \in \{0,\ 1\}$ for all $k = 1, \ldots, K$, $t \in [0, t_f]$ such that

$$\sum_{k=1}^{K} q_{k,[t_{i-1},t_i)}(t) = 1.$$

Let $F(x)$ be a vector field associated with the given switched-mode system (1). We next introduce a "sequencing control vector" $q_{[t_{i-1},t_i)}(t)$associated with the time interval $[t_{i-1}, t_i)$

$$\begin{aligned}&F(x) := \{f_1(x), \ldots, f_K(x)\},\\ &q_{[t_{i-1},t_i)}(t) := (q_{1,[t_{i-1},t_i)}(t), \ldots, q_{K,[t_{i-1},t_i)}(t))^T\end{aligned}$$

and use the simplified notation $q^i(t) \equiv q_{[t_{i-1},t_i)}(t)$. Note that every component $q_{k,[t_{i-1},t_i)}(t)$ of $q^i(t)$ corresponds to a particular mode $f_k(x)$ of (1) on the concrete time interval $[t_{i-1}, t_i)$. Therefore, the sequencing control vector (a vector function) $q^i(\cdot) \in \mathscr{Q}_K$, where

$$\mathscr{Q}_K := \left\{ \theta : [0, t_f] \to \{0,\ 1\}^K \;\middle|\; \sum_{k=1}^{K} \theta_k(t) = 1 \right\},$$

represents the schedule of modes for every time instant $t \in [0, t_f]$.

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