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Structural synthesis of parallel manipulators with coupling sub-chains

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ABSTRACT

A conventional parallel manipulator uses serial kinematic chains or the legs to support a single moving platform. The movement of the platform may be further constrained by adding interconnections among its legs to form the so-called coupling sub-chains. The introduction of coupling sub-chains to the kinematic structures of parallel manipulators, while allowing for improved performance and functionality, presents also significant challenges in the structural synthesis of the resulting manipulators. This paper presents a systematic method for the structural synthesis of parallel manipulators with coupling sub-chains. The contracted graphs in connection with Euler's equation are constructed based on permutation and combination, and the isomorphism is identified. Then, fractionation and simplification process of the contracted graph of the resulting manipulators are analyzed. In the second half of the paper, a screw theory based method for constraint synthesis of kinematic chains is developed to synthesize various legs and coupling sub-chains. Examples were provided to demonstrate the efficacy of the advocated method.

1. Introduction

Traditionally, a parallel manipulator is defined as a closed-loop mechanism in which the moving platform, or end-effector, is connected to the base by at least two independent serial kinematic chains [1,2]. Typically, the actuators on the connecting chains, or the legs, are mounted at or near the fixed base. In the past decades, most of the parallel manipulators have been reported or published consistent with the above traditional definition. In order to improve performances and functionality of parallel manipulators, in recent years, researchers have studied novel parallel manipulators that do not fit well the traditional definition. For example, parallel manipulators with coupling sub-chains [3–5], and parallel manipulators with configurable platform [6–8]. We call these parallel manipulators as generalized parallel manipulators. Generalized parallel manipulator is a multi-DOF (degrees of freedom) single or multiple close-loop planar or spatial mechanism, and all the actuators of the manipulator are mounted on or near the fixed base. One or more links of the manipulator can be selected as the output element according to mobility and functionality requirements of the manipulator. The structural synthesis approaches for traditional parallel manipulators cannot be directly applied to parallel manipulators with coupling sub-chains. The aim of this paper is to develop a systematic structural synthesis method for parallel manipulators with coupling sub-chains.

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In general, the kinematic design of mechanisms and manipulators follows a two-step process, structure synthesis and dimensional synthesis. In the past decades, the structural synthesis of novel planar or spatial parallel manipulator has received significant attention in the research community as it is the essential aspect in the creative design process [9,10]. Huang and Li [11] put forward a general methodology to synthesize lower-mobility parallel manipulators with applications to the design of a number of manipulators. Kong et al. [12] used constraint screws to the synthesis of parallel manipulators. Hervé [13] and Li et al. [14] presented a class of novel parallel manipulators which satisfy the algebra structures of displacement group. Fang and Tsai [15] investigated the structure synthesis of 4-DOF and 5-DOF serial kinematic chains and parallel manipulators. Gogu [16] explored the translational parallel manipulators by the theory of linear transformations. Ge et al. [17] proposed the unified task driven synthesis approach to design planar parallel mechanisms. Yang et al. [18] put forwarded the novel characteristic set and equations for serial mechanisms and parallel mechanisms to design spatial mechanisms. Gao et al. [19,20] presented a concept of G*f* set to construct parallel mechanisms. Xie et al. [21] summarized the type synthesis of parallel kinematic mechanisms by using the Grassmann line geometry method.

So far, however, much of the efforts on structural synthesis has focused on the traditional parallel manipulators such that the moving platform is connected to the fixed base with two or more serial kinematic chains [22]. In general, parallel manipulators with coupling sub-chains have the potential to offer higher accuracy and stiffness. In addition, parallel manipulators with coupling sub-chains may output special motion for specified requirements [7,20]. Campos et al. [23] proposed the Assur groups and synthesized kinematic chains and generalized mechanisms by using the chain groups. Zeng and Fang [5] developed the structural synthesis of multi-loop limb structures by using displacement group theory. Zoppi et al. [24] presented a novel parallel manipulator which has interconnected-chains and can be used to manufacture revolute surfaces. Hao et al. [25–27] proposed a class of generic symmetrical manipulators with coupling sub-chains for the actuation isolation/decoupling purpose. In these manipulators, the actuation legs are added to implement actuation isolation and the cross-axis coupling motions are introduced. Shen et al. [28] explored the methodology of parallel embedment to construct a class of 6-DOF hybrid mechanisms. Ding et al. [3,29] systematically proposed a method for dealing with structural synthesis and mobility analysis of two-layer two-loop hybrid mechanisms. Bałchanowski [30] presented the method of synthesizing spatial manipulators and created a complete set of spatial manipulator solutions. Besides, the contracted graph obtained by contracting all binary vertices is an effective and basic tool for type synthesis of spatial manipulators [31]. Vucina and Freudenstein [32] introduced graph theory into the structural synthesis of closed loop mechanisms. Lu and Leinonen [33] addressed contracted graphs from associated linkages via topology matrices. Gogu [9] developed the structural synthesis of spatial mechanisms utilizing the evolutionary and morphological method.

The aim of this paper is to develop a systematic structural synthesis method for parallel manipulators with coupling sub-chains. Accordingly, the organization of this paper is as follows: In Section 2, based on permutation and combination method, the contracted graphs in connection with Euler's equation are constructed. The representation which signifies the connective relationship of basic links is presented and the isomorphism is identified. Section 3 covers the fractionation and simplification process of contracted graphs of the resulting manipulators. Moreover, the screw theory based method for constraint synthesis of kinematic chains is deduced in section 'Screw Theory Based Method for Constraint Synthesis'. Then, the combination of different types of limbs which are used to construct parallel manipulators with coupling sub-chains is proposed. Finally, a number of novel parallel manipulators with coupling sub-chains are synthesized and conclusions are drawn.

2. Representation of contracted graphs

The structural synthesis and topological structures of close-loop mechanisms can be represented by corresponding contracted graphs. In the contracted graphs, each note or vertex represents a link and each line or edge denotes a joint, all links are drawn on a circle and the links are connected with each other by different edges. The basic links are distinguished by the degree of the vertex that is denoted as the number of edges in the vertex.

In accordance with the equivalent conditions [34], a mechanism can be denoted by its associated linkage, which is made up of various types of basic links: binary B, ternary T, quaternary Q, pentagonal P, or hexagonal H links [33]. The formula for calculating DOF of the associated linkage of parallel manipulators with coupling sub-chains is derived from the modified Kutzbach–Grübler formula [35], in accordance with equivalent conditions, as below [36]

$$M = 6(N - J - 1) + J + v - \zeta \tag{1}$$

where M is the DOF of the manipulator, N signifies the total number of basic links in the manipulator, J represents the number of all kinematic pairs, v is the number of redundant constraints and ζ is the number of passive DOF. Let n_k ($k = 1, 2, 3\cdots$) denote the number of links having k joints, then we have

$$N = n_2 + n_3 + n_4 + \cdots = \sum_{k=2} n_k \tag{2}$$

$$J = \frac{1}{2}(2n_2 + 3n_3 + 4n_4 + \cdots) = \frac{1}{2}\sum_{k=2} k \cdot n_k \tag{3}$$

One of the distinguishing features of a parallel manipulator with coupling sub-chains is the availability of additional independent loops. To this end, Euler's formula [2,37] can be employed to account for the number of the independent