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Sparse Control for Dynamic Movement Primitives

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Abstract:
This paper describes the use of spatially-sparse inputs to influence global changes in the behavior of Dynamic Movement Primitives (DMPs). The dynamics of DMPs are analyzed through the framework of contraction theory as networked hierarchies of contracting or transversely contracting systems. Within this framework, sparsely-inhibited rhythmic DMPs (SI-RDMPs) are introduced to both inhibit or enable rhythmic primitives through spatially-sparse modification of the DMP dynamics. SI-RDMPs are demonstrated in experiments to manage start-stop transitions for walking experiments with the MIT Cheetah. New analytical results on the coupling of oscillators with diverse natural frequencies are also discussed.

Keywords: Dynamic movement primitives, central pattern generators, contraction analysis, nonlinear oscillators, legged locomotion, networked systems.

1. INTRODUCTION

There is a growing body of evidence that motor primitives may form the basis for a rich set of sensorimotor skills in humans and animals (Mussa-Ivaldi et al., 1994; Bizzi et al., 1995; Rohrer et al., 2004; Hogan and Sternad, 2012). From walking to grasping, the composition of primitive attractors could provide robustness as behaviors are generalized and recycled from past experience. Primitives may, in a sense, represent a compression of experience, capturing accumulations of knowledge that may be drawn on to simplify online control. This use of motor primitive techniques in biological systems would be well supported by the underlying nature of evolutionary change. Indeed, evolution necessarily proceeds through the accumulation of stable intermediate states (Simon, 1962), building upon existing functional frameworks through stably layered complexity.

The use of dynamic movement primitives (DMPs) (Ijspeert et al., 2012) has sought to embody these principles for the development of sensorimotor skills in robotics. Dynamic movement primitives are systems of coupled ordinary differential equations that represent a target attractor landscape for robot motion. The attractor landscapes can be learned through demonstration (Ijspeert et al., 2002) or crafted through manual design. The landscapes of DMPs may represent attractors for a wide range of rhythmic and discrete movements (Schaal, 2006; Pastor et al., 2009).

Rhythmic DMPs are closely related to the mimicry of biological Central Pattern Generators (CPGs) (Marder and Bucher, 2001) within robotics (Ijspeert, 2008). A hallmark of CPGs in biological systems is that a low-dimensional set of inputs can be used to orchestrate coordinated patterns of high-dimensional oscillatory motor control signals. Stable oscillations of Andronov-Hopf oscillators (Chung and Slotine, 2010) have been employed for pattern generation in bioinspired control of locomotion in air (Chung and Dorothy, 2010) and water (Seo et al., 2010). Stable phase oscillators (Ajallooeian et al., 2013b) have been supplemented with sensorimotor feedback to stabilize quadrupedal locomotion (Ajallooeian et al., 2013a; Barasuol et al., 2013). Across these results, low-dimensional inputs are capable of smoothly reshaping high-dimensional target behaviors for dynamic machines.

Despite the popularity of DMP/CPG frameworks, analysis of couplings between primitive modules has largely been lacking in the literature. Contraction analysis (Lohmiller and Slotine, 1998) provides modular stability tools which may help to guide the architecture of more flexible and robust DMP/CPG frameworks. A preliminary analysis of discrete DMPs through contraction theory was provided in (Perk and Slotine, 2006), with new analysis in this paper using transverse contraction theory (Manchester and Slotine, 2014b; Tang and Manchester, 2014). Contracting systems are characterized by an exponential forgetting of initial conditions, providing a notion of stability without committing in advance to a particular trajectory. Such a notion is desirable from a practical standpoint, as success in situations form grasping a cup to running down a cliff are hardly characterized by unique solutions.

The composition of primitive contracting systems suggests a promising approach for robust online synthesis from offline knowledge (Lohmiller and Slotine, 1998; Perk and Slotine, 2006; Slotine and Lohmiller, 2001; Manchester et al., 2015). As we will see, contracting systems provide an abstraction of their performance, namely a contraction metric, contraction rate, and associated contraction region, which compactly characterize properties and ro-

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bustness of composition. Contraction metrics, which guide online control, might be learned offline through drawing on experience, or through evolution, enabling application in systems beyond the limitations of current control synthesis tools. Experiments in learning stable attractors from demonstration (Khansari-Zadeh and Billard, 2011) can be cast as convex problems through a contraction viewpoint (Ravichandar and Dani, 2015). This suggests that a notion of motor stability resembling contraction could guide a form of sensorimotor learning with favorable convergence.

These burgeoning extensions of contraction analysis offer an opportunity to understand and extend seemingly-complex robot control frameworks. The main contributions of this paper are to provide an analysis of Dynamic Movement Primitives (DMPs) within the framework of contraction and to introduce a new functional tool for DMPs through spatially-sparse inhibition. Contraction analysis of DMPs provides new results related to scaling primitives in space through general diffeomorphisms, on the stability of rhythmic DMPs in general networked combinations, and robustness to parameter heterogeneity in coupled oscillators. Aside from using low-dimensional inputs to shape rhythmic high-dimensional behavior, we show that DMPs can be globally shaped through spatially-sparse modification to the DMP vector fields. This extension, which we call sparsely-inhibited DMPs (SI-DMPs) is used to manage start/stop transitions for phase oscillators in locomotion experiments with the MIT Cheetah robot.

The paper is organized as follows. Section 2 presents DMPs and draws on commonality across varied implementations in the literature. Section 3 provides preliminaries on contraction analysis, which are then used to analyze the stability of DMPs. Section 4 builds on this analysis with an extension to sparsely inhibit Rhythmic DMPs. Section 5 presents the validation of these results to inhibit oscillations that drive locomotion in a walking gait for the MIT Cheetah robot. A short discussion and concluding remarks are provided in Section 6.

2. DYNAMIC MOVEMENT PRIMITIVES

Dynamic movement primitives (Ijspeert et al., 2012) are systems of ordinary differential equations which can be used to generate target kinematic behaviors for robotic systems. While there are many implementations of DMPs within the literature, a single DMP (i.e. not coupled to any others) is generally structured as a hierarchy of three separate systems: a reference system, canonical system, and transformation system (Ijspeert et al., 2012). We begin by providing examples of these systems in the literature, and then describe their common general properties.

2.1 Discrete (Point-To-Point) Motion Primitives

Discrete DMPs encode point-to-point motions, shaping both the behavior of the kinematic targets, as well as transients along the approach. Letting g represent a goal configuration, the state $(y, \dot{y}, x) \in \mathbb{R}^3$ of a point-to-point DMP may be chosen to evolve as (Ijspeert et al., 2012)

$$\tau\ddot{y} = k(g - y) - b\dot{y} + f(x) \tag{1}$$

$$\tau\dot{x} = -\alpha_x x \tag{2}$$

where $k \in \mathbb{R}^+$, $b \in \mathbb{R}^+$ provide spring and damper values for a desired attractor towards the goal $g \in \mathbb{R}$, $\tau \in \mathbb{R}^+$ a temporal scaling factor and $f(x)$ a forcing function. The variables $(y, \dot{y})$ encode a position and velocity for the output of the DMP, while x is a phasing variable which smoothly decays to zero. The forcing function $f(x)$ can shape the transient behavior via phased-based forcing through Gaussian basis functions

$$f(x) = \frac{\sum_i \Phi_i(x) w_i}{\sum_i \Phi_i(x)} x, \quad \Phi_i(x) = \exp\left(-\frac{(x - c_i)^2}{2\sigma_1^2}\right) \tag{3}$$

It is common to learn weights w_i for these forcing functions through demonstration (Ijspeert et al., 2012), with learning accomplished through least-squares methods. In order to increase smoothness of the output, reference systems may be employed to filter external commands, for instance with an externally provided goal $g_{ext}(t)$

$$\dot{g} = \alpha_g(g_{ext}(t) - g)\,. \tag{4}$$

Beyond translating the goal, adjustable attractor landscapes through spatial and time-based scaling have been sought as key features within DMPs (Ijspeert et al., 2012).

Consistent with the literature (Ijspeert et al., 2012) (1) is called a *transformation system* while (2) is called a *canonical system*. The role of the canonical system is to provide a notion of phase, while the transformation system uses the phase to shape the attractor landscape. Rhythmic primitives generalize this framework through the inscription of oscillations into the canonical system.

2.2 Rhythmic Motion Primitives

Letting $\mathbf{x} = (x_1, x_2) \in \mathbb{R}^2$, represent a new canonical system state, a choice for rhythmic DMP dynamics is

$$\tau\ddot{y} = k(g - y) - b\dot{y} + f(\mathbf{x}) \tag{5}$$

$$\tau\dot{x}_1 = \quad \omega x_2 + \rho(r^2 - x_1^2 - x_2^2)x_1 \tag{6}$$

$$\tau\dot{x}_2 = -\omega x_1 + \rho(r^2 - x_1^2 - x_2^2)x_2 \tag{7}$$

The $\dot{\mathbf{x}} = \mathbf{f}_\mathbf{x}(\mathbf{x})$ dynamics in (6)-(7) are a stable Andronov-Hopf oscillator at radius r.[1] The forcing function $f(\mathbf{x})$ provides phase-dependent forcing through von Mises bases

$$f(\mathbf{x}) = \frac{\sum_i \Phi_i(\theta(\mathbf{x}))\mathbf{w}_i^T}{\sum_i \Phi_i(\theta(\mathbf{x}))}\mathbf{x}, \ \Phi_i(\theta) = \exp\left(\frac{\cos(\theta - \theta_i) - 1}{2\sigma_1^2}\right)$$

where the angle of $\mathbf{x}$ denoted $\theta(\mathbf{x}) = \mathrm{atan2}(x_2, x_1)$. Filters similar to (4) may be added to smoothly shape references, such as the nominal center of oscillation g or the oscillation amplitude r, in response to changes in external reference.

2.3 Commonalities

Across these examples, and across the literature, there is a great deal of commonality in the varied implementations of DMPs. As highlighted previously, we can typically decompose each DMP into three separate subsystems:

$$\dot{\mathbf{r}} = \mathbf{f}_\mathbf{r}(\mathbf{r}, \mathbf{r}_{ext}) \qquad \text{(Reference System)} \tag{8}$$

$$\dot{\mathbf{x}} = \mathbf{f}_\mathbf{x}(\mathbf{x}, \mathbf{r}) \qquad \text{(Canonical System)} \tag{9}$$

$$\dot{\mathbf{y}} = \mathbf{f}_\mathbf{y}(\mathbf{x}, \mathbf{y}, \mathbf{r}) \qquad \text{(Transformation System)} \tag{10}$$

where $\mathbf{r} \in \mathbb{R}^{n_r}$ the reference state, $\mathbf{r}_{ext} \in \mathbb{R}^{n_r}$ an external command, $\mathbf{x} \in \mathbb{R}^{n_x}$ the canonical (phase) state, and $\mathbf{y} \in \mathbb{R}^{n_y}$ the transformed output. Within the categorizations

[1] This definition differs slightly from previous canonical systems in polar coordinates (r, θ) (Ijspeert et al., 2012). A stable limit cycle for $\mathbf{x}$ simplifies analysis for rhythmic DMPs here.