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IFAC PapersOnLine 50-1 (2017) 2665–2670

Contraction Based Tracking Control of Autonomous Underwater Vehicle

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Abstract: This paper discusses the incremental stability of an underwater vehicle using the contraction theory. Stability analysis considered in vehicle dynamics of a simple and a more advanced model has the ability to constructing the controller to track the desired trajectory of underwater vehicle position. The natural contracting behavior of an underwater vehicle is ensured for a more advanced model of vehicle to derive contracting exponentially stable controller. The tuning parameters of the controller have selected analytically from the incremental stability analysis using contraction theory. The controller design is restricted to parametric-strict-feedback form to develop an incremental backstepping design technique. In this paper, proposed method afford to construct a controller in a recursive way and it enforces incremental exponential stability of an underwater vehicle and not just global asymptotic stability.

Keywords: Incremental stability, tracking control, exponential stability, contraction theory, underwater vehicle.

1. INTRODUCTION

Controlling the horizontal position of an underwater vehicle changing at the bottom of the sea is of great importance for precise maneuvering. Real-time accurate position measurements of underwater vehicles are facilitated by the latest advances in sensing technology of vehicle position and velocity (Marco and Healey (1998),Whitcomb et al. (1998)). In order to precisely control the low-speed trajectory of an underwater vehicle, these advances in position sensing are greatly backup to the controller design. But, our understanding of the dynamics of the bladed thrusters and vehicle are principally limited to the control application while these dynamics commonly used to actuate dynamically positioned marine vehicles (Healey et al. (1994),Whitcomb and Yoerger (1999a)). This research work focuses on the design of position controller with the use of a simple (Fossen (1994)) as well as more advanced models of the underwater vehicle (Whitcomb and Yoerger (1999b)).

The nonlinear control systems design in a recursive way has been shown great interest for the past so many years. The backstepping method is one of the nonlinear technique, which provides a recursive design procedure in a systematic manner for the systems transformable into parametric strict feedback form (Khalil (1996)). In this method, suitable Lyapunov functions are constructed at each stage to confirm the stability of each subsystem (Khalil (1996)). Lyapunov stability theory is widely explored to the design of nonlinear control system. On the contrary, contraction theory is used as latest method to the convergence analysing of nonlinear systems (Lohmiller and Slotine (1998),Lohmiller and Slotine (2000)). It is treated as an incremental form of stability since contraction theory offers a tool for the stability analysis of nonlinear system trajectories with respect to each other (Lohmiller (1999)). Such a property of nonlinear systems is a stronger property than global exponential convergence to a single trajectory.

In the literature, Lyapunov-based Uniform Global Exponential Stability (UGES) of system dynamics is proven and extended its application to the underwater vehicle with the nonlinear contracting observer (Jouffroy and Fossen (2004)). Likewise, the application of contraction analysis has been pursued to derive the nonlinear observer by several authors (Majeed and Kar (2013),Jouffroy (2003),Jouffroy and Lottin (2002b),Jouffroy and Slotine (2004)). The contraction theory framework offers significant advantages that it 1) eliminates the need to know the equilibrium point of system dynamics as it works for an incremental convergence of trajectories, and 2) does not require difficult task of the selection of a suitable Lyapunov function for stability analysis. As a result, stability analysis does not require a skillful simplification to show negative definiteness of the derivative of a Lyapunov function. For a particular class of nonlinear systems, an integrator backstepping procedure in contraction framework is introduced in (Jouffroy and Lottin (2002a)). In the light of such framework, UGES control law is derived in this paper using contraction theory to track the desired trajectory of an underwater vehicle position. The main contributions of this paper are

(1) Contraction theory based an incremental backstepping control law is derived to tracking the desired position of an underwater vehicle. The simple and more advanced models of underwater vehicles are adopted for this purpose. In a practical point of view, this result is considered to be highly useful as it permits for the systematic design of contracting exponentially stable controllers.

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10.1016/j.ifacol.2017.08.472

(2) The selection of tuning parameters of the controller is obtained analytically as a by-product of the incremental stability analysis in contraction theory. The natural contracting behavior of an underwater vehicle is established for a simplified UGES control law in the case of a more advanced model of vehicle.
(3) Contracting control law is tested through the simulation studies for the simple and more advanced model of underwater vehicle.

The following Section 2 describes the design of integral backstepping position control of an underwater vehicle with its numerical simulation in section 3. Finally, the conclusions are given in section 4.

2. POSITION CONTROL OF AUTONOMOUS UNDERWATER VEHICLE

We are considered in turn a simple and more advanced models of underwater vehicle to derive the UGES control law based on the contraction theory in order to track the desired trajectory of vehicle position. Contraction theory is a tool to analyze the convergence between two arbitrary system trajectories. If the trajectories of the perturbed system return to their nominal behavior with an exponential convergence rate, such a nonlinear system is said to be contracting. The main result of contraction theory is given in Lemma 1 (Lohmiller and Slotine (1998),Lohmiller and Slotine (2000),Jouffroy and Slotine (2004)).

Lemma 1: For a given system $\dot{x} = f(x,t)$, there exists a scalar $\xi > 0, \forall x, \forall t \geqslant 0$ such that $\frac{\partial f}{\partial x} \leqslant -\xi I < 0$ or $\frac{1}{2}\left(\frac{\partial f}{\partial x} + \frac{\partial f}{\partial x}^T\right) \leqslant -\xi I < 0$, any trajectory which starts in a ball of constant radius centred about a given trajectory and contained at all a time in a contraction region, remains in that ball and converges exponentially to the given trajectory. Moreover, global exponential convergence to this given trajectory is guaranteed if the whole state space region is contracting.

In the case of feedback combinations of two systems, results of contraction theory are extended as follows: Consider the two systems possibly of different dimensions whose dynamics are given by

$$\begin{aligned} \dot{x}_1 &= f_1(x_1, x_2, t) \\ \dot{x}_2 &= f_2(x_1, x_2, t) \end{aligned} \tag{1}$$

If these systems are connected in feedback combination, we can write virtual displacements in transformed domain as

$$\frac{d}{dt}\begin{bmatrix} \delta z_1 \\ \delta z_2 \end{bmatrix} = \begin{bmatrix} F_1 & G \\ -G^T & F_2 \end{bmatrix} \begin{bmatrix} \delta z_1 \\ \delta z_2 \end{bmatrix} \tag{2}$$

Then the augmented system is contracting if and only if the separated plants are contracting.

2.1 A Simple Model of Underwater Vehicle

A simple model to describe the speed of underwater vehicle is proposed by Healey and Marco (1992) from the nonlinear underwater vehicle equations of motion (**?**). The speed equations of an underwater vehicle with actuator can be written in the form of parametric-strict feedback as follows :

$$\begin{cases} \dot{q} = v \\ \dot{v} = -\dfrac{d}{m}|v|v + \dfrac{\eta}{m} \\ \dot{\eta} = -\dfrac{\eta}{T} + \dfrac{u}{T} \end{cases} \tag{3}$$

where m is the mass of underwater vehicle in kg, d is vehicle drag coefficient in Ns^2/m^2 and T is the time constant of actuator. The state vector of (3) consists of vehicle position q, vehicle velocity v and vehicle torque η. Control signal u is acting as the commanded input to the actuator. In the position control of underwater vehicle, control objective is to design a feedback control law u so that the vehicle position $y = q$ converges to a desired trajectory of y_d with all other signals remaining bounded. The following lemma derives the resulting control law in contraction framework.

Lemma 2: Given an underwater vehicle system (1), the actual vehicle position $q(t)$ will track a reference trajectory $y_d(t)$ if control law u is given by

$$\begin{aligned} u = {} & T\left(\frac{-z_1}{m} - (k_1 + k_2)\, z_2 + \frac{(z_2 + \beta_2)}{T}\right) \\ & + T\left(d\left(z_1 + \beta_1 - |z_1 + \beta_1|\right)(z_1 + \beta_1)\right) \\ & + T\left[d\left(z_1 + \beta_1 + |z_1 + \beta_1|\right) - k_1 m\right] \\ & \times \left[-e_1 + \frac{z_2}{m} + (1 - k_1)\, z_1\right], \; k_1 > 0, k_2 > 0 \end{aligned} \tag{4}$$

where auxiliary variables z_1 and z_2 are defined as $z_1 = v - \beta_1(q)$ and $z_2 = \eta - \beta_2(q, z_1)$ respectively with $\beta_1(q) = -[q - y_d] + \dot{y}_d$ and $\beta_2(q, z_1) = d|v|v - k_1 z_1 m + \ddot{y}_d m$.

Proof: Defining tracking error $e_1 = y - y_d$ and selecting auxiliary variables as $z_1 = v - \beta(q)$, where $\beta(q)$ the virtual control input, makes the first subsystem in (3) contracting with reference to q. Thus, the dynamics of subsystem can be written as

$$\dot{q} = z_1 + \beta_1(q) \tag{5}$$

The virtual displacement of this system can be represented in differential framework as

$$\delta\dot{q} = \delta z_1 + J_{11}\delta q \tag{6}$$

where Jacobian matrix J_{11} is represented by

$$J_{11} = \frac{\partial}{\partial q}\left[\beta_1(q)\right] \tag{7}$$

This Jacobian J_{11} is Uniformly Negative Definite (UND) in nature by the selection of $\beta_1(q)$ as follows

$$\beta_1(q) = -[q - y_d] + \dot{y}_d = -e_1 + \dot{y}_d \tag{8}$$

Thus, $z_1 = v + e_1 - \dot{y}_d$ and hence the first subsystem can be reduced into error dynamics of vehicle position as

$$\begin{aligned} \dot{e}_1 &= \dot{q} - \dot{y}_d = v - \dot{y}_d \\ &= -e_1 + z_1 \end{aligned} \tag{9}$$

By taking time derivative of z_1 and using (3) and (5), we get

$$\begin{aligned} \dot{z}_1 &= \dot{v} + \dot{e}_1 - \ddot{y}_d \\ &= -\frac{d}{m}|v|v + \frac{\eta}{m} - e_1 + z_1 - \ddot{y}_d \end{aligned} \tag{10}$$

Define new virtual control input $\beta_2(q, z_1)$ to make (10) contracting w.r.t. z_1. The feedback interconnection of subsystems are ensured in (5) and (10). Defining new auxiliary variable $z_2 = \eta - \beta_2(q, z_1)$, (10) can be represented as

$$\dot{z}_1 = -e_1 - \frac{d}{m}|v|v + \frac{z_2}{m} + z_1 + \frac{\beta_2}{m} - \ddot{y}_d \tag{11}$$

Thus, the virtual displacement in differential framework for the combination of the first two subsystems (9) and (11) can be represented as

$$\begin{bmatrix} \delta\dot{e}_1 \\ \delta\dot{z}_1 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -1 & J_{22} \end{bmatrix} \begin{bmatrix} \delta e_1 \\ \delta z_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/m \end{bmatrix} \delta z_2 \tag{12}$$