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Stochastic contraction based online estimation of second order wiener system

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ABSTRACT

Wiener system is a block oriented model, having a linear time-invariant dynamic system followed by a memory-less nonlinearity. To design a stochastic estimator for online estimation of Wiener system of second order, this paper utilizes differential mean value theorem and the results of stochastic contraction theory. The asymptotic convergence of proposed estimator is derived by using contraction theory related to semi-contracting systems. The boundedness and convergence of the parameter and state estimates have been shown analytically. The introduced method has potentials to estimate accurately states and parameters of Wiener model simultaneously from the noisy output of the system and unknown structure of nonlinearity. Numerical simulation of the stochastic estimator is presented to justify the claim by considering the two examples of the real world system with an additive measurement noise.
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1. Introduction

The difficulty of Wiener model identification is the interaction of the linear and nonlinear blocks as shown in Fig. 1. The system identification task is challenging for Wiener model because the intermediate signal, $y(t)$ is usually not available for measurement. Thus identification process has to depend only on observations of input u(t) and output $y_n(t)$. A standard procedure to identify the nonlinear Wiener system is to transform the measured output signal with the inverse of nonlinearity in order to linearize the system with the assumptions that the static nonlinearity is known and invertible. To invert a nonlinear block, a high gain observer has been used to estimate the states of the system together with the construction of parameter estimator [1]. The conventional linearization technique through inversion can destroy the signal to noise ratio of the data when the measurement noise affects the output. Thus, recursive parameter estimation is derived from a parametric Wiener model using prediction error method without applying inversion of the nonlinearity [2]. The basic idea is that if the linear block can be reliably identified then the intermediate signal $y(t)$ recovered, and identification becomes much easier. Therefore, several algorithms are proposed in the literature to identifying the linear transfer function of a block oriented Wiener system [3–7].

While there are several methods for identifying Wiener models proposed in the literature, the most dominant of these is to parameterize the linear and the nonlinear blocks [8], and then estimate the parameters from data by minimizing an output-error criterion [9]. A difficulty with this approach is that it cannot handle the process noise which is disturbing the Wiener model. A wide range of techniques for estimating the mixed linear and nonlinear Wiener system is reported in [4,5,10,11]. Recently, Wills and B. Ninness [12,13] expanded Wiener model in more general structure, and carried out its benchmark study with commonly used nonlinearities. For the general disturbance case, maximum likelihood method is efficiently implemented using the implication of Bussgang's theorem [14]. The nonlinear static block of preload and dead zone are appearing in the form of a discontinuous piece-wise linear function, which can be represented as a linear combination of known smooth functions (polynomials) [10,15]. In such a case, identification of the nonlinear block is reduced to estimating unknown parameters. Recently, auxiliary model identification idea and hierarchical identification principle are used to identify the block oriented model by considering the polynomial model for static nonlinearity [16]. Since the Wiener model blocks are assigned to model the behavior of specific parts of the real world system, there is a wide range of its applications which are reported in [14,17–21,25]. They have been applied to both natural phenomena, such as the pH control process [17], and the man-made devices like control valves [18] and the power amplifiers [19–21].

The main purpose of this paper is to design a Wiener estimator in the form of Itô stochastic differential equations (SDEs) to estimate states and parameters of a Wiener model from the noisy measurement. To analyze the convergence property of the estimator, differential mean-value theorem and results of stochastic contraction theory are used. There are two main factors motivating this

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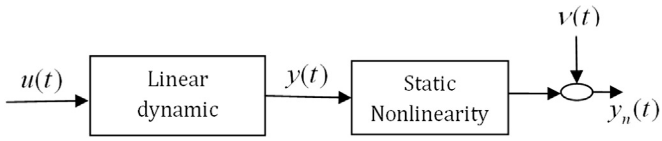

Fig. 1. Block diagram of the Wiener model, the input $u(t)$ and the output $y_n(t)$ are measurable, but not the intermediate signal $y(t)$. The static nonlinearity is assumed to be memory-less.

approach. First is the non-availability of a static nonlinearity structure of the system in practice. Second is the attraction of scientific contributions of 1) contraction theory in the investigation of globally exponential convergence of noisy trajectories [22,23] and 2) differential mean value theorem to observer design [24]. The observer design using differential mean value theorem has the advantage of wide range of operating condition for a globally bounded Jacobian system [24]. This is due to the non-convexity of the associated prediction error criterion. The approach presented here can provide an effective estimation of Wiener model since the contraction theory consists of incremental exponential stability property, and the differential mean value theorem are used to design unknown gains of a proposed Wiener estimator.

In this paper, we adopt the Lipschitz-like approach as in [26] to estimate the states and the parameters of a system based on the continuous observation of its noisy measurement, but the convergence analysis of the estimator has been analyzed using the stochastic contraction theory. The contraction theory based stability analysis is defined as an incremental convergence between the two arbitrary trajectories [27–30]. The exponential stability of certain nonlinear systems is easy to be shown in contraction theory. Since the framework of contraction theory works on incremental analysis of neighboring trajectories, it eliminates the need to know the equilibrium point of the dynamical system. The selection of a suitable Lyapunov function is also not required for the stability analysis in contraction framework. The approach given in this paper utilizes differential mean value theorem and the results related to semi-contracting systems to prove the convergence of the proposed Wiener estimator. The main contributions of this paper are

1. Stochastic contraction based an estimator is proposed to estimate the states and the parameters of the Wiener model from the noisy output. Itô SDEs are used for the convergence analysis of the stochastic estimator.
2. The tuning parameters of Wiener estimator have selected analytically from the result of the contraction theory based incremental stability analysis. The differential mean value theorem is utilized to express the nonlinear error dynamics as a convex combination of known matrices with time varying coefficients.

The rest of the paper is organized as follows. The problem formulation is described in Section 2. To prove the convergence analysis of the proposed estimator, mean value theorem for a bounded Jacobian system and some useful results related to the stochastic contraction theory are also described in Section 2. Numerical results are presented in Section 3 to justify the claim of the Wiener estimator and Section 4 gives the conclusions.

2. Problem formulation

Consider a nonlinear dynamical system of the form

$$\dot{x} = f(x, t) \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is a state vector and $f: \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$ is a continuously differentiable nonlinear function. A stochastically perturbed system of the nominal system (1) represented by an Itô stochastic differential equation (SDE) with the measurement equation is given by

$$\begin{aligned} dx &= f(x, t)dt + B(x, t)dW, \; x(0) = x_0 \\ y &= h(x, t) + \sigma\nu(t) \end{aligned} \tag{2}$$

The existence and uniqueness of a solution to (2) are realted to the following conditions (p. 106, [31])

$$\begin{cases} \exists\, L_1 > 0, \; \forall\, t, \; \forall\, x_1, x_2 \in \mathbb{R}^n: \\ \left\| f(x_1, t) - f(x_2, t) \right\| + \left\| B(x_1, t) - B(x_2, t) \right\| \le L_1 \|x_1 - x_2\|, \\ \exists\, L_2 > 0, \; \forall\, t, \; \forall\, x_1 \in \mathbb{R}^n: \\ \left\| f(x_1, t) \right\|^2 + \left\| B(x_1, t) \right\|^2 \le L_2\left(1 + \|x_1\|^2\right) \end{cases} \tag{3}$$

Where $B: \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^{n\times m}$ is a matrix-valued function, $y(t) \in \mathbb{R}^m$ is the measurement, $h(x, t): \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^m$, σ is the noise intensity and zero mean m dimensional white noise is considered as measurement noise $\nu(t)$ which is defined as $dW = \nu(t)dt$, where W is standard m dimensional Wiener process.

2.1. Stochastic contraction theory

The basic results on stochastic contraction theory is stated here.[1] For this purpose, the stochastic observer for estimating the states of system (2) can be written as

$$d\hat{x} = f(\hat{x}, t)dt + \mathrm{G}(\hat{x}, t)\left(y(x, t) - y(\hat{x}, t)\right)dt + \mathrm{G}(\hat{x}, t)\sigma dW \tag{4}$$

where $\mathrm{G}(\hat{x}, t)$ is the gain of the observer. The following assumptions are made to explain the stochastic contraction theory.

Assumption 1. Noise free version of the stochastic observer (4) is contracting in the identity metric, with contraction rate λ.

$$\text{i. e. } \forall\, x,\, t \ge 0, \quad \lambda_{\max}\left(\frac{\partial f(x, t)}{\partial x} - \mathrm{G}(\hat{x}, t)\frac{\partial y(x, t)}{\partial x} \right)\Bigg|_{x=\hat{x}} \le -\lambda \tag{5}$$

where $\lambda_{max}(\,.\,)$ is the maximum eigenvalue of $(\,.\,)$.

Assumption 2. $\mathrm{trace}\left(\left(\mathrm{G}(\hat{x}, t)\sigma\right)^T \mathrm{G}(\hat{x}, t)\sigma \right)$ is upper bounded by a constant C, i.e

$$\mathrm{trace}\left(\left(\mathrm{G}(\hat{x}, t)\sigma\right)^T \mathrm{G}(\hat{x}, t)\sigma \right) \le C \tag{6}$$

Lemma 1 (*stochastic contraction [22]*). *If Assumptions 1 and 2 are true, then estimated trajectories of the stochastic observer (4) from a noisy measurement exponentially converge to the trajectories of (2) with convergence rate λ within a bound of $\frac{C}{2\lambda}$,*

$$\text{i. e. } \forall\, t \ge 0, \quad \mathrm{E}\left(\left\| \hat{x}(t) - x(t) \right\|^2 \right) \le \frac{C}{2\lambda} + \left\| \hat{x}_0 - x_0 \right\|^2 e^{-2\lambda t},$$

$$\text{where } \lambda = \inf_{x,t} \left| \lambda_{\max}\left(\frac{\partial f(x, t)}{\partial x} - \mathrm{G}(\hat{x}, t)\frac{\partial y(x, t)}{\partial x} \right)_{x=\hat{x}} \right|, \quad \text{and}$$

$$C = \sup_{t\ge 0} \mathbf{trace}\left(\left(\mathrm{G}(\hat{x}, t)\sigma\right)^T \mathrm{G}(\hat{x}, t)\sigma \right)$$

Proof. Refer to [22].

Remark 1. The construction of stochastic observer is possible for the stochastic system if its noise free part is contracting, and the gain $\mathrm{G}(\hat{x}, t)$ of this observer can be obtained from the Assumptions 1 and 2. The estimation error of the stochastic observer is upper bounded by a constant of $\frac{C}{2\lambda}$, which is a function of the noise

[1] Basics of stochastic contraction theory are stated in appendix for more clarity.

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