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Journal of Sound and Vibration 304 (2007) 831–844
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

Detecting weak sinusoidal signals embedded in a non-stationary random broadband noise—A simulation study

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Received 19 January 2007; received in revised form 6 March 2007; accepted 18 March 2007
Available online 2 May 2007

Abstract

In the present study, a simple numerical function is proposed for use together with the time–frequency analysis in the detection of very weak sinusoidal signals embedded in a non-stationary random broadband background noise. Its performance is studied through the use of two numerical examples. It is found that the present method enables good recovery of the sinusoidal signals and the instants of their initiations even when the signal-to-noise ratio is down to −17 dB.

1. Introduction

Signal detection technique has wide applications in both physics and engineering (for instance, Refs. [1,2]). For a modernized heavily serviced building, the early detection of fault signals from rotary machineries, such as the chillers, pumps, motors, etc., is crucial to its smooth operation [3]. Similar situation appears in electricity power plants. The magnitudes of the fault signals are usually very small when the faults are in their initial stages. Though the faults will usually result in abnormal spectral peaks in the machine vibration signals [4], their detection is not straightforward in the initial stage because of the presence of much stronger background signals or noises, which can be stationary or non-stationary.

Owing to the importance of signal detection in the machine health monitoring/diagnosis process, many analysis approaches have been introduced in the past few decades. The short-time Fourier transform (STFT) [5] appears to be a very obvious choice. The use of the Wigner distribution and wavelet transforms for analyzing non-stationary signals has also been studied [6,7]. Tang [8] investigated the performance of STFT and the harmonic wavelets [9] in retrieving parameters of exponential decaying pulses. Besides, the use of time series techniques for signal analysis has been examined. For example, Zhan and Jardine [10] analyzed gear faults using the auto-regression, while Chan et al. [11] investigated decaying sinusoidal pulses using the stochastic volatility approach. However, though there has been much effort made in improving signal detection and analysis, many of the proposed approaches fail to give satisfactory results when the signal-to-noise ratio (S/N) is close to 0 dB or goes negative, which is the case in the early development of a fault in the

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0022-460X/$ - see front matter
doi:10.1016/j.jsv.2007.03.024

building services equipment. Li and Qu [12] investigated the use of the chaotic oscillator for weak signal detection, but their method requires knowledge on the characteristics of the signals to be detected.

The focus of the present study is on enhancing the detection of weak signals embedded in a stronger non-stationary signal/noise. A numerical treatment to the original signal made up of the weak sinusoidal signal and the non-stationary noise, which can facilitate a better retrieval of the weak signal after the time–frequency procedure, is proposed. Its performance is examined through two illustrative examples.

2. Theoretical considerations

The target of the present study is to establish a method to detect effectively a sinusoidal signal under a poor S/N. This section describes the development of the method. The spectral technique will be used, but it is believed that the method also works for wavelet transforms. In the foregoing analysis, all parameters are non-dimensional. Also, Ω denotes random noise with vanishing mean and magnitude bounded between ± 1 in the foregoing discussions. These signals are generated using the software MATLAB.

It is understood that the extraction of a very weak sinusoidal signal from a background noise consists of many different frequency components is difficult. One needs a method to down-weigh the effect of the background noise in the spectral calculation. A method which tends to reduce the magnitudes of the time series data but preserves the periodicity of the sinusoidal signal will be very useful. It is expected a pulse-like train will be created if one can preserve the peaks and troughs of the sinusoidal signal while the other parts of the signal are down-weighted together with the noise in the very ideal case.

2.1. Power spectral density of a square pulse train

An infinite square pulse train is perhaps the most fundamental signal in building services engineering other than the sinusoidal wave. It is given by the expression

$$x(t) = A \sum_{i=-\infty}^{\infty} (-1)^i [U(t - it_o - (t_o - \Delta)/2) - U(t - it_o - (t_o + \Delta)/2)], \tag{1}$$

where U is the unit step function, and A and Δ are real constants which fix the magnitude and duration of each pulse in $x(t)$, respectively. The period is $2t_o$. Fig. 1 illustrates an example of the train with $A = 1$, $t_o = 10$ and $\Delta = 6$. The power spectral density of x, $P(x, \omega)$, is [13]

$$P(x, \omega) = \lim_{T\to\infty} \frac{2}{T^2} \left| \int_0^T x(t) e^{-j\omega t}\, dt \right|^2, \tag{2}$$

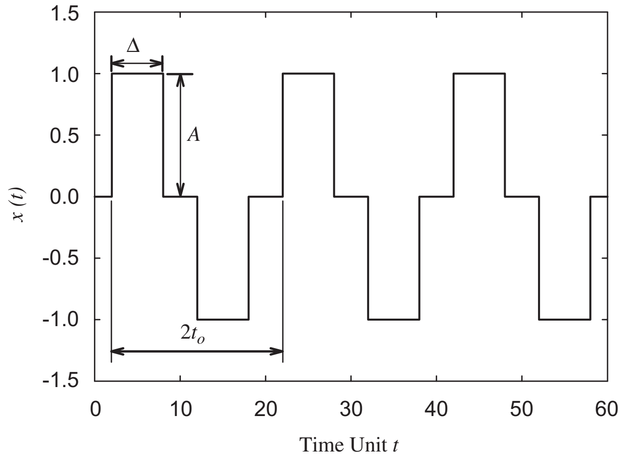

Fig. 1. Example of a square pulse train and the nomenclatures. $\Delta = 6$, $A = 1$, $t_o = 10$.