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Innovative Applications of O.R.

Equilibrium joining strategies in batch service queueing systems

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A R T I C L E I N F O

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We consider strategic customers in a Markovian queue with batch services. We derive customer equilibrium strategies, regarding the joining/balking dilemma, in two cases with respect to the information provided upon arrival, unobservable and observable. In contrast to models with single services, a customer's decision to join induces both positive and negative externalities to other customers. This fact leads to an intricate mixture of Follow-The-Crowd and Avoid-The-Crowd behavior and possibly multiple equilibrium strategies. Moreover, we discuss the effects of the two levels of information and the batch size on the strategic behavior of the customers and on the overall social welfare. Finally, we present several numerical experiments that reveal important differences in the strategic behavior of customers in batch service systems, in juxtaposition to single service systems.

1. Introduction

The study of strategic customer behavior in queueing systems has attracted the interest of many investigators, since it enables a more accurate representation and quantification of the operation of a system. The vast majority of the papers in this area, however, concerns single service systems, where the customers are served one by one. On the other hand, queueing systems with batch services occur quite frequently in practice. For example, a shuttle in an airport transportation facility departs as soon as all its seats are occupied. In museums/exhibitions, the visitors are served in groups and a guided tour starts only when a sufficient number of visitors have been accumulated to form a group. In boat cruises, a trip begins when the capacity of the boat has been reached. In amusement parks, the rides are performed in cars with a certain number of passengers and customers have to wait until a car is full, before they can start the ride. Given their wide range of applications and the limited relevant work on systems with batch departures (see the literature review below), it seems important to extend this thread of game-theoretic research to batch service queueing systems.

On top of that, the consideration of batch service systems seems particularly important, as there are important differences in strategic customer behavior between single and batch service systems. The objective of the present paper is to study these differences. For instance, in most cases, in a single service system, an observed or anticipated high congestion level constitutes a discouraging factor for arriving customers that face the dilemma of whether to join or balk. In some cases, though, the presence of a large number of customers may encourage more customers to enter the system. For example, this is the case when the service speed increases with the number of customers, or in systems that pass to a sleep mode after an idle time and resume service only after a certain amount of customers has been accumulated. A similar situation is also observed whenever congestion is associated with high quality. However, these cases are considered rather exceptional and we can fairly say that the general rule for single service systems is the Avoid-The-Crowd (ATC) situation, i.e. the larger the number of customers that prefer to enter a system, the less willing a tagged customer becomes to join it. In contrast to single service systems, intuitive considerations suggest that batch service systems do not exhibit an ATC behavior. Indeed, in the latter systems, the waiting time of a tagged customer depends both on the processing times of waiting complete batches and on the completion time of his own incomplete batch. Therefore, an increase of the arrival rate does not induce only the negative effect of increasing the mean overall processing time for the waiting complete batches. It also shortens the mean completion time of his own batch by future arrivals.

These arguments show that the influence of the number of waiting customers in a batch service system on a tagged customer splits in two factors: the number of complete batches and the number of customers in the incomplete batch found upon arrival. It is clear that the more the complete batches, the less willing is the tagged customer to join, i.e. in this dimension, the system behaves like the single service systems and exhibits an ATC behavior.

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On the other hand, a large number of customers in the incomplete batch incites a tagged customer to join and we have a Follow-The-Crowd (FTC) situation. The coexistence of these two opposite effects implies that the strategic behavior of the customers is an intricate mixture of ATC and FTC and deserves further analysis. In particular, it seems important to investigate the trade-off of these effects. Another interesting question is to clarify which is the prevalent effect, depending on the operational and reward/cost parameters of a system.

In conclusion, the study of strategic customer behavior in batch service systems is not only interesting per se, as it provides important managerial insights, but has also a wide range of applications. Therefore, it is highly classified for further investigation. In addition, it is also important to assume the administrator's perspective, as well as pricing problems and the control of the batch size.

1.1. Literature review

Providing customers the option to decide whether to join or balk allows us to treat the whole situation as a non-cooperative game among them, since they act with the objective of maximizing their individual expected utility. In terms of Game Theory, the main objective of such an analysis is the derivation of the possible equilibrium patterns that result from the decentralized control that induce the customers.

The study of customer strategic behavior regarding the joining/balking dilemma was initiated in the pioneering paper of Naor (1969), who studied the $M/M/1$ queue, assuming an observable system, i.e. upon arrival a customer is informed about the queue length and then makes his decision. Edelson and Hildebrand (1975) complemented this study by studying the corresponding unobservable system. Since then, there is a growing number of papers that deal with the game-theoretic analysis of the balking behavior of customers in variants of the M/M/1 queue, see e.g. Burnetas and Economou (2007), Guo and Zipkin (2007), Hassin (2007), Economou and Kanta (2008a, 2008b), Guo and Hassin (2011, 2012), Wang and Zhang (2013), Shone, Knight, and Williams (2013), Zhang, Wang, and Liu (2013), Zhou, Lian, and Wu (2015), Ziani, Rahmoune, and Radjef (2015) and Dimitrakopoulos and Burnetas (2016). It is interesting to notice that most of these studies are still focused on simple variants of the $M/M/1$ queue. However, although the performance analysis of the above systems is in most cases straightforward, the game-theoretic analysis is substantially more difficult. Indeed, the analysis requires the quantification of such a system under a general customer strategy and usually leads to Markovian models that are intractable or difficult to solve, even for systems of $M/M/1$ type. Moreover, the computation of equilibrium strategies rarely is carried out in closed form, but rather via an algorithmic procedure. The monographs (Hassin & Haviv, 2003; Stidham, 2009) and Hassin (2015) summarize the main approaches and several results in this broad area of the economic analysis of queueing systems.

On the other hand, the literature on the strategic customer behavior in service systems with some kind of batch departures is very scarce and falls in three classes of queueing models: Systems with zero service times, systems with infinite servers and clearing systems. The last class refers to systems where all present customers are removed simultaneously. Two subcategories lie in it: systems legitimate for transportation facilities that remove all waiting passengers from a station and systems subject to catastrophic events. We now briefly present the main findings in each class.

Hassin and Haviv (2003) (Section 1.5) considered a shuttle model, where customers arrive according to a Poisson arrival process and decide whether to go to a bus station, where buses arrive according to a renewal process, or to a shuttle service that departs whenever the number of waiting passengers reaches the transporter's capacity. This is a batch service model with zero service times, in the sense that a batch of passengers leaves the system immediately after its formation. In other words, there is no queue of batches waiting to be served, but rather the study of the waiting time of a customer boils down to the computation of the completion time of his own batch. The authors make the simplifying assumption that there is always an available shuttle, ready for departure, as soon as the required number of customers is completed. Under this assumption they obtain equilibrium and socially optimal strategies in two informational cases, according to whether the arriving customers observe the number of waiting commuters at the shuttle terminal or not.

Systems with infinite servers for batch services have been studied in a sequence of papers. Calvert (1997) introduced a seminal model with an $M/M/1$ queue and an $M/G^{(M)}/\infty$ queue, where M stands for the batch size, and studied equilibrium routing customer strategies. The main motivation of this work was to provide a queueing model version of a transportation system which exhibits the Downs–Thomson paradox. More specifically, he proved that increasing the service rate does not necessarily improve the system performance, but may cause an increase in the expected delay for some range of the parameters of the model. Afimeinounga, Solomon, and Ziedins (2005, 2010) and Chen, Holmes, and Ziedins (2012) considered important generalizations of this model in various directions. In Afimeinounga et al. (2005), the authors made a thorough analysis of the model under probabilistic routing and studied the existence, uniqueness and stability of the equilibrium strategies. Using coupling arguments, the same authors investigated the state-dependent counterpart of the model in Afimeinounga, Solomon, and Ziedins (2010). A similar system with two parallel batch service queues and state-dependent and probabilistic routing was studied in Chen et al. (2012). Other interesting generalizations in this thread of research are reported in Afimeinounga (2011) and Pai and Cheng (2012).

Economou and Manou (2013) and Manou, Economou, and Karaesmen (2014) studied two models for the strategic customer behavior in clearing systems, where a service facility removes all present customers periodically. This situation occurs in modeling transportation stations, where the passengers are removed at the successive visits of a transportation facility. The authors determined the equilibrium joining strategies for the customers under various levels of information and discussed the influence of the various parameters and distributions of the models on the customer strategic behavior.

The study of the strategic behavior of customers in queueing systems with catastrophes was initiated in Boudali and Economou (2012, 2013). In these systems, the service is provided to customers one by one. However, customers may be also forced to abandon the system in batches, without being served, when a catastrophic event occurs (e.g. an unexpected failure of the system). The authors studied the customer joining behavior in such systems, when there is a reward for the regular service, but also a compensation whenever a customer is removed from the system due to a catastrophe.

Apart from the aforementioned studies, it seems that the strategic customer behavior in queueing systems with some kind of batch departures has received only limited attention. In particular, to the best of our knowledge, there are no game-theoretic studies for systems with regular batch services, in which the batches form a queue. However, the consideration of strategic customers in such systems seems more appropriate in the service and manufacturing sectors. The objective of the present work is to fill this gap of the literature. Therefore, it seems legitimate to begin the study with the simplest one: the $M/M/1$ queue with single arrivals and batch services of fixed size K (also known as the $M/M^K/1$ queue). From a modeling perspective, this model bears some similarities