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Exploring experiences for assisting primary pre-service teachers to extend their knowledge of student strategies and reasoning

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ABSTRACT

Exploring how students learn mathematics and what types of tasks promote mathematical reasoning may assist pre-service teachers to develop their pedagogical content knowledge (PCK) for teaching. This study reports on an experience of a cohort of Australian primary (elementary) pre-service teachers, destined to become specialist mathematics teachers associated with the topics of measurement and geometry. The first author presented a lesson with Year 5/6 students related to geometric reasoning and calculating the size of angles. The lesson was launched with no instructions and students were expected to attempt a problem without using a protractor or help from the teacher. The pre-service teachers, classroom teacher and second author were non-participant observers. Findings suggest course experiences provided an opportunity to extend pre-service teachers' knowledge of how students learn and their knowledge of lesson structure and PCK whilst also considering how students might learn to reason and solve a challenging mathematical task.

1. Introduction

There is a need for teacher education programs that prepare pre-service teachers to extend their knowledge of how students learn mathematics, including how to attend to students' development of their reasoning skills. Recent approaches to mathematical learning suggest that students should be provided with rigorous experiences that encourage them to rely on their knowledge in different contexts and unfamiliar situations (Sullivan et al., 2013) and that enhancing students' learning depends on the knowledge of the teacher (Koellner et al., 2007). Many pre-service teachers may not have experienced practices that emphasise reasoning as a type of approach, when learning mathematics at school (Maher, Palius, Maher, Hmelo-Silver, & Sigley, 2014). Therefore, it is important that teacher education courses consider experiences and types of task types that model and promote mathematical reasoning as well as pedagogical approaches, that encourage students to reason and problem solve for themselves.

1.1. Knowledge for teaching

The skills and knowledge required by a primary mathematics teacher are complex. Over the past thirty years there has been a significant focus on developing frameworks that assist with deepening our understanding of the categories used to describe the types of knowledge required for mathematics teaching (e.g., Ball, Thames, & Phelps, 2008; Chick et al., 2006; Rowland, Turner, Thwaites, & Huckstep, 2009). Ball et al., (2008) domains of Mathematical Knowledge for Teaching, built on the seminal work of Shulman (1986,1987) and included two categories of subject matter knowledge and pedagogical content knowledge (PCK):

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- Subject matter knowledge: *common content knowledge*, *specialised content knowledge* and *horizon content knowledge;*
- PCK: *knowledge of content and students, knowledge of content and teaching and knowledge of curriculum*

It is worth highlighting that mathematical reasoning has not been included within any of the teacher knowledge frameworks within this review of literature. However, reasoning is one of the four proficiencies of the Australian Curriculum (Australian Curriculum Assessment and Reporting Authority (ACARA), 2016) and embedded within the National Council of Teachers of Mathematics (NCTM) standards (NCTM, 2000) and a type of knowledge required to support students' mathematical learning.

1.2. Reasoning and teaching

Recent reviews of the research literature call for developing the types of classroom communities that promote reasoning (e.g., Larkin, Grootenboer, & Lack, 2016; Mueller et al., 2014). Maher et al., 2014 suggested that providing primary (elementary) pre-service teachers with opportunities to work with students in classrooms may assist them to recognise students' emerging reasoning. Students demonstrate mathematical reasoning when they justify strategies and reach conclusions, plus reasoning should be consistently developed within many contexts (Australian Curriculum Assessment and Reporting Authority (ACARA), 2016; NCTM, 2000). Furthermore, pre-service teachers need to learn the knowledge and skills of how to develop students' reasoning skills as it is one of the four proficiency strands and fundamental to the Australian mathematics curriculum.

Within the Australian curriculum for mathematics, geometry and reasoning can be taught together, and geometric reasoning is introduced at Year (Grade) 3 (Australian Curriculum Assessment and Reporting Authority (ACARA), 2016). Geometric reasoning is developed through critical thinking, logical argument and spatial reasoning when exploring geometric relationships (Australian Association of Mathematics Teachers, 2014). Sullivan (2011) in an Australian Education Review suggested that teachers may have difficulty knowing how to support students' reasoning skills when teaching. Furthermore, geometry education within primary school classrooms should be encouraging students to move beyond naming and sorting shapes by extending their spatial reasoning (Sinclair & Bruce, 2015).

Since the introduction of reasoning within the Australian curriculum in 2011, recent studies have focused on teachers' perceptions of reasoning including professional learning designed to develop teachers' approaches for teaching geometry and spatial reasoning (e.g., Sullivan et al., 2015). Loong, Vale, Bragg, and Herbert (2013) reported on primary teachers' perceptions of reasoning, and recommended designing teacher professional learning programs that assist teachers to plan worthwhile tasks for developing students' reasoning knowledge. They defined a framework listing seven categories used to describe teachers' perceptions of mathematical reasoning: *thinking, communicating thinking, problem solving, validating thinking, forming conjectures, using logical arguments for validating thinking, forming conjectures, using logical arguments for validating conjectures* and *connecting aspects of mathematics* (Herbert, Vale, Bragg, Loong, & Widjaja, 2015). Other researchers suggest that open ended tasks, including the approach to the lesson structure can create an opportunity for students to develop their mathematical reasoning (Sullivan, 2011; Sullivan et al., 2015; Sullivan & Davidson, 2014) and geometric reasoning (Livy, Ingram, Holmes, Linsel, & Sullivan, 2016).

1.3. Lesson structure that promotes student reasoning

A larger research project, the Encouraging Persistence Maintaining Challenge (EPMC) project was designed to help teachers to engage students in reasoning and productive struggle through attempting challenging mathematical tasks (e.g., Sullivan et al., 2015). These tasks (including learning tasks) have been designed and described as cognitively demanding and pose problems the students should find challenging because at the beginning of the lesson they will not yet know how to solve them (Sullivan & Davidson, 2014; Sullivan et al., 2016). The learning tasks developed for the professional learning in 2015 were designed for teaching geometric reasoning (mainly angles) and teachers participating in the project were encouraged to use a three-phase lesson structure when using the challenging tasks (Sullivan et al., 2015).

First, during the *launch* phase, students are expected to attempt the learning task by themselves. Second, during the *explore phase,* the teacher monitors and selects students to present their working out. After students make an initial attempt at the task, they may be provided with an *enabling prompt* as a sub-task, if they are struggling (Sullivan, Zevenbergen, & Mousley, 2005). Other students who quickly or easily completed the learning task may be provided with an *extending prompt*. In the *summarise phase* the teacher pauses the lesson and invites pre-selected students to share their thinking, reasoning and strategies. As shown in Fig. 1 the phases are cyclical and may occur more than once during the lesson (Sullivan et al., 2015).

During the EPMC project, teachers were encouraged to provide minimal instruction when launching the tasks because teachers sometimes 'over-explain' when introducing a task thereby dramatically reducing the opportunity for productive struggle (Roche & Clarke, 2014; Russo, 2015). After the *launch phase* students were expected to attempt the task first by themselves and teachers were encouraged to let the students struggle. Taking risks, making mistakes and struggling are important to developing students' mathematical thinking and leads to productive classrooms where students can work on complex problems (Boaler, 2016; Warshauer, 2014).

As part of the *summarise phase* the teachers considered the five practices for orchestrating productive mathematics discussion (Sullivan et al., 2015). The five practices include: *anticipating* students' likely responses, *monitoring* responses, *selecting* students to share their strategies and thinking, *sequencing* the order students will share their strategies and *connecting* the responses with the key mathematical ideas that students are learning (Smith & Stein, 2011). The *summarise phase* is important for highlighting student strategies and reasoning and may assist other students who are working out the problem (Livy et al., 2016).