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Uncertainty quantification in structural dynamic analysis using two-level Gaussian processes and Bayesian inference

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ABSTRACT

A probabilistic framework for efficient uncertainty quantification in structural dynamic analysis is presented. This framework is built upon the combination of two-level Gaussian processes emulator and Bayesian inference technique. The underlying idea is to employ the two-level Gaussian processes emulator to integrate together small amount of high-fidelity data from full-scale finite element analysis and large amount of low-fidelity data from order-reduced analysis to improve the response variation prediction. As component mode synthesis (CMS) is adopted in order-reduced modeling, we then utilize the improved response variation prediction on modal characteristics to update the CMS model to facilitate the efficient probabilistic analysis of any responses of concern. The effectiveness of this framework is demonstrated through systematic case studies.

1. Introduction

Real-world structures are subject to uncertainties due to material imperfection, machining tolerance, and assemblage error, etc. A deterministic analysis that is based on the nominal model inevitably over- or under-estimates the response of an actual structure. Incorporating uncertainties in modeling and measurements into the dynamic analysis of a structure that evaluates its functional performance has obvious importance [1]. Ideally, in analyzing a structure, variations in parameters of concern can be approximately defined in terms of probabilities, based on which Monte Carlo-based analysis is to be performed to generate the response variations. However, when a single run of numerical simulation or physical experiment is too expensive, such Monte Carlo-based analysis will be impractical [2,3].

One school of thoughts is to create a more economical model to reduce the computational cost of each single dynamic analysis. In fact, efficient and accurate order-reduction techniques for structural dynamic analysis have long been pursued along with the development of the finite element method [4–6]. The essence of many order-reduction techniques is to represent a full-scale finite element model in a subspace via an appropriate transformation basis. A representative class of order-reduction methods in structural dynamic analysis is called the component mode synthesis (CMS) approach, where the transformation basis is constructed by keeping only a few lowest-order modes of individual substructures and also including some additional compensating modes for recovering information in the modes that are truncated [7–10]. The CMS is a traditional and popular order-reduction approach in structural dynamics and vibration analysis. The efficiency of CMS is

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obvious, as it renders a model that requires only a few modes of individual substructures. This computational cost reduction is attractive in uncertainty analysis using Monte Carlo sampling-based analysis [11,12]. In addition, it should be noted that CMS is a natural way of employing experimentally acquired data directly, as the modal information of substructures that is obtained experimentally can be integrated into the reduced-order model [13–15].

In the realm of statistical analysis, meanwhile, there has been growing interest in developing statistical meta-models to replace the whole Monte Carlo simulation, by using for example the so-called Gaussian processes [16]. The underlying idea of Gaussian processes is to extend the multivariate Gaussian distribution from a finite dimensional space to an infinite dimensional space [17,18]. DiazDelao and Adhikari [19,20] reported the usage of such an approach in structural dynamic analysis, where Gaussian processes were employed as emulators to approximate frequency responses (mean and probability of mean) of simple structures. Fricker et al. [21] systematically studied the uncertainty quantification of frequency response function of a testbed structure, and demonstrated that the Gaussian processes could give more accurate predictions than traditional linear and quadratic response surfaces. Xia and Tang [22] implemented the Gaussian processes in the uncertainty quantification of a spatially periodic structure that has inherently high sensitivity of dynamic response with respect to uncertainties.

Fundamentally, both the CMS-based order reduction and the Gaussian processes approach are subject to errors. On one hand, the CMS order-reduction can allow more efficient generation of first principle-based response prediction; on the other hand, the truncation of higher-order modes in each substructure generally results in error as compared with a full-scale finite element analysis. In the case that a CMS model is used in Monte Carlo simulation, the resultant sample very probably has errors that distort the statistical measures, e.g., mean and standard deviation, in response characterization. The usage of Gaussian processes as emulators, while being able to yield extremely fast analysis by using only a few data points, inevitably causes error due to its inference nature, even if all the data points involved can be considered accurate (i.e., coming from a full-scale finite element model in numerical simulation). Increasing the amount of data points in the Gaussian processes can mitigate the inference error. However, high-accuracy data are difficult to obtain in practical applications due to computational cost issue, which is indeed the reason that CMS-type order reduction has been adopted even in deterministic analysis. Moreover, if we opt to use the CMS-based results as data points for Gaussian processes, the sample will always inherit the errors in these data points no matter how well Gaussian processes works.

Indeed, along with the advancement of Gaussian processes approach, a technique of utilizing data at multiple resolutions for inference has been studied, which is referred to as hierarchical Gaussian process [17,23]. It was suggested that the combination of data with different fidelities for Gaussian process emulation can maintain both prediction accuracy and efficiency [17]. As such, one enabling element of this research is to formulate a two-level Gaussian processes emulator for response variation prediction in structural dynamic analysis, which is capable of unleashing the full power of both CMS and Gaussian processes approaches while addressing the concerns and existing issues mentioned above. In this new approach, the CMS based order-reduction is used to generate large amount of low-fidelity, low-cost, first principle-based data. With the large amount of such data, the Gaussian processes emulator can avoid those errors associated with the inference procedure. Meanwhile, with the introduction of a few high-fidelity data, we can correct the error of the low-fidelity data inherited from the order-reduction procedure.

Existing two-level Gaussian processes mainly focus on improving the quality of a specific set of output data based upon corresponding, available data at multiple levels. In the context of structural dynamic analysis, however, one is usually interested in *a series* of data sets, e.g., various responses at different locations under excitations which themselves may vary. Conceptually, it would be much more desirable if one could come up with an efficient and accurate, probabilistic, low-dimension model that can directly predict various response variations at arbitrary location(s) of the structure under varying excitations. Because of the involvement of CMS order-reduced model here, this concept can be realized. A CMS model retains the underlying physics of the original, nominal structure. What needs to be done here is to use response variation data with sufficient fidelity to update it in a probabilistic manner. Therefore, the objective of this research is to establish a new framework that embeds the two-level Gaussian processes into the probabilistic model updating procedure to produce a probabilistic CMS model. The Bayesian inference naturally fits the model updating requirement here due to its intrinsic advantages [24–27]. The essence of Bayesian inference is to establish a probabilistic model to correct the prior beliefs based on the evidences. In the new framework, we plan to use the modal information of the entire structure obtained through two-level Gaussian processes as the evidences, and update the probabilistic distributions of key parameters of the CMS model, i.e., the modal information of individual substructures used in the model. The resultant probabilistic model can then eventually be used to perform response variation analysis. To accelerate the Bayesian inference to fit large-scale structural dynamic analysis, we integrate the Markov Chain Monte Carlo (MCMC) optimization scheme [28,29] that employs a significantly reduced number of analysis runs. Collectively, the integration of these algorithms leads to a new computational framework of response variation prediction that features both high efficiency and high accuracy.

The rest of the paper is organized as follows. In Section 2, we provide details of the new computational framework. We start from outlining a representative CMS procedure, and then present the two-level Gaussian processes and the Bayesian inference that connect the CMS with full-scale finite element predictions to facilitate probabilistic model updating. In Section 3, we report a systematic case study to illustrate the implementation and the improved response variation predictions for structural dynamic analysis. Concluding remarks are summarized in Section 4.