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Multifactor CES general equilibrium: Models and applications

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ABSTRACT

Sector-specific multifactor CES elasticities of substitution and the corresponding productivity growth are jointly measured by regressing the growth of per-factor cost shares against the growth of factor prices. We use linked input-output tables for Japan and the Republic of Korea as the data source for factor price and cost shares in two temporally distant states. We then construct a multisectoral general equilibrium model using the system of estimated CES unit cost functions and evaluate the economy-wide propagation of an exogenous productivity stimulus in terms of welfare. Further, we examine the differences between models based on *a priori* elasticities such as Leontief and Cobb-Douglas.

1. Introduction

In this study, we jointly measure the multifactor CES elasticity of substitution with the productivity growth for multiple industrial sectors by way of two temporally distant cross-sectional data (i.e. linked input–output tables). When we apply the multifactor CES unit cost function, we discover that industry-specific elasticities can be estimated by regressing the growth of per-factor cost shares against the growth of factor prices. We also discover that industry-specific productivity growth can be measured via the intercept of the regression line. Consequently, we make use of the linked input–output tables in order to observe the cost shares and price changes spanning over two periods for multiple industrial sectors.

The two-input constant elasticity of substitution (CES) function was first introduced by Arrow et al. (1961), Uzawa (1962), McFadden (1963) later showed that elasticities were still unique for the case of more than two factor inputs. Empirical analyses concerning the measurement of CES elasticities (e.g. McKibbin and Wilcoxen, 1999; van der Werf, 2008; Koesler and Schymura, 2015) have been based upon time series data, while embedding nest structures into the two-input CES framework conforming to the work by Sato (1967), to handle elasticities between more than two factors of production. The number of factors and thus, of estimable elasticities, can nevertheless be reduced, depending on the availability of time series data. Since we are interested in constructing a multisector general equilibrium model that calls for multifactor production functions, we take the advantage of an alternative approach by exploiting cross-sectional data.

When a multisectoral general equilibrium model is established, assessments can be made of the arbitrary productivity shock resulting from technological innovation in terms of the welfare gained. Previous studies in this regard have assumed constant and uniform unit elasticity (Klein, 1952–1953, Saito and Tokutsu, 1989), or have used empirically estimated elasticities in Translog or multistage (nested) CES functions with a highly aggregated and thus limited number of substitutable factors. Examples include works by Kuroda et al. (1984) and Tokutsu (1994) and many of the works concerning CGE models, such as studies by Böhringer et al. (2015), Go et al. (2016). In contrast, our approach allows us to construct an empirical model of multifactor production with different elasticities of substitution among many (over 350) industrial sectors. Moreover, this approach allows us to prospectively portray the ex post technological structure following any given exogenous productivity shock and to account for welfare in terms of economy-wide input–output performances.

We measure the welfare changes attributed to the exogenous productivity change by SCS (social cost saved), i.e. the difference in the total primary factor inputs required to net produce a fixed amount of final consumption, given the productivity change. We find theoretically that SCS will be positive (primary factor inputs will always be saved) in every sector if the exogenous productivity is improving and, vice versa, under a system with uniform CES elasticity less than unity, which is inclusive of Cobb–Douglas and Leontief systems. Hence, conversely, such a law may not necessarily hold for the case of CES systems with non-uniform elasticities, as verified by the empirical analysis of SCS using the estimated multifactor CES system.

The remainder of this paper is organised as follows. In the next section, we introduce the basics of multifactor CES elasticity and productivity

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growth estimations, and apply the protocol to linked input–output tables for Japan and the Republic of Korea, which have sufficient capacity in terms of the degrees of freedom of the regressions. In Section 3, we replicate the current technological structure as the general equilibrium state of a system of empirically estimated multifactor CES functions; further, we trace how that structure is transformed by exogenous productivity stimuli. Section 4 provides concluding remarks.

2. The model

2.1. Multifactor CES functions

A constant-returns multifactor CES production function of an industrial sector (index j omitted) has the following form:

$$y = zf(\mathbf{x}) = z\left(\sum_{i=0}^{n} \lambda_i^{\frac{1}{\sigma}} x_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

where y denotes the output and x_i denotes the ith factor input. Here the share parameters are assumed to maintain $\lambda_i > 0$ and $\sum_i \lambda_i = 1$, while the elasticity of substitution $\sigma \geq 0$ is subject to estimation. Also, we are interested in measuring the growth of productivity, i.e. $\Delta \ln z$, where Δ represents temporally distant differences.

Displayed below is the unit cost function compatible with the multifactor CES production function:

$$c = z^{-1}h(\mathbf{w}) = \frac{1}{z}\left(\sum_{i=0}^{n} \lambda_i w_i^{\gamma}\right)^{1/\gamma}$$

where c denotes the unit cost of the output and w_i denotes the ith factor price. Here, we use $\gamma = 1 - \sigma$ for convenience. The cost share of the ith input a_i can be determined, in regards to Shephard's lemma, by differentiating the unit cost function:

$$a_i = \frac{\partial c}{\partial w_i}\frac{w_i}{c} = \lambda_i (zc/w_i)^{-\gamma} \tag{1}$$

By taking the log of both sides, we have

$$\ln a_i = \ln \lambda_i - \gamma \ln z + \gamma \ln(w_i/c)$$

As we observe two temporally distant values for cost shares (${a_i}^0$ and ${a_i}^1$), factor prices (${w_i}^0$ and ${w_i}^1$) and unit costs of outputs as prices ($c^0 = w^0$ and $c^1 = w^1$) reflecting perfect competition, we find two identities regarding the data:

$$\ln a_i^0 = \ln \lambda_i - \gamma \ln z^0 + \gamma \ln(w_i^0/w^0) + \epsilon_i^0$$
$$\ln a_i^1 = \ln \lambda_i - \gamma \ln z^1 + \gamma \ln(w_i^1/w^1) + \epsilon_i^1$$

where we assume that ${\epsilon_i}^0$ and ${\epsilon_i}^1$ are identically and normally distributed disturbance terms. Subtraction results in the main regression equation as follows:

$$\Delta \ln a_i = -\gamma \Delta \ln z + \gamma \Delta \ln(w_i/w) + \epsilon_i \tag{2}$$

Here, the disturbance term $\epsilon_i = \epsilon_i^0 - \epsilon_i^1$ is identically and normally distributed, so that one can estimate γ and $\Delta \ln z$ via a simple linear regression Eq. (2). That is, by regressing the growth of per-factor cost shares, i.e. $\Delta \ln a_i$ on the growth of relative prices or $\Delta \ln(w_i/w)$, the slope gives the estimate of γ, while the intercept gives the estimate of $-\gamma \Delta \ln z$. Also, note that λ_i can be calibrated via Eq. (1), as long as we have the estimate for γ.

2.2. The data and estimations

A set of linked input–output tables includes sectoral transactions in both nominal and real terms. Since real value is adjusted for inflation, in order to enable comparison of quantities as if prices had not changed, and since nominal value is not adjusted, we use a price index to convert nominal values into real values. That is, if we standardise the value of a commodity at the reference state as real, its nominal (unadjusted) value at the target state relative to the reference state equals the price index called a deflator. Naturally, the 1995–2000–2005 linked input–output tables for both Japan (MIAC, 2011) and Korea (BOK, 2015) include per-factor deflators (395 factors for Japan and 350 factors for Korea) spanning the fiscal years recorded. These linked input–output tables, however, do not include deflators for primary factor (i.e. labour and capital) and therefore, we use the quality-adjusted price indexes compiled by JIP (2015) for Japan and by KIP (2015) for Korea in order to inflate the primary factor inputs observed as nominal values.

Hence, observations for both the dependent variables (cost shares as input–output coefficients a_{ij}) and independent variables (price ratios w_j/w_i) for estimating Eq. (2) become available with sufficient capacity in terms of degrees of freedom, as we verify that there are $n + 1$ inputs; namely, $i = 0, 1, \ldots, n$ and n outputs; namely, $j = 1, \ldots, n$ for an input–output table. In particular, we use the 2000 and 2005 input–output coefficient matrices from the three-period linked input–output tables as the data for the cost share growth (i.e. $\Delta \ln a_{ij}$) and, since we set the reference state at year 2000, the five-year growth of output-relative factor prices becomes simply the log difference between deflators; that is,

$$\Delta \ln w_i/w_j = \ln p_i/p_j$$

where p_i denotes the deflator for commodity i in year 2005 with respect to year 2000.

Fig. 1 displays the estimated CES elasticity (i.e. $\sigma_j = 1 - \gamma_j$) with respect to the statistical significance of γ_j, i.e. the slope of the regression Eq. (2) in terms of the P-value in Japan. Fig. 2 is the version for Korea. Note that CES elasticities are statistically significant (P-value <0.1) for 176 out of 395 sectors for Japan, whereas 166 sectors are significant out of 350 sectors in Korea. The results of the estimations are summarised in the Appendix, Tables A1 and A2, for Japan and Korea, respectively. These tables are confined to sectors whose slopes ($\gamma_j = 1 - \sigma_j$) of the regression (2) are statistically significant and we indicate the level of significance by *** (0.01 level), ** (0.05 level) and * (0.1 level), along with the estimated elasticities. Note that we accept the null hypothesis (i.e. $\gamma_j = 1 - \sigma_j = 0$) for sectors with a statistically insignificant slope and, in that event, the average elasticity, i.e. $\sum_{j=1}^{n} \sigma_j/n$ is 1.32 for Japan and 1.39 for Korea. Alternatively, if we accept all the elasticity estimates, regardless of statistical significance, the average elasticity is 1.46 for Japan and 1.52 for Korea.

These multifactor CES elasticities are comparable to other estimates in the literature. The GTAP (2016) substitution elasticities for intermediate inputs, which are broadly employed in CGE studies (e.g., (Álvarez-Martínez and Polo, 2012; Antimiani et al., 2015)) range from 0.20 to 1.68, while those among internationally traded goods (i.e. Armington elasticities) are generally larger, ranging from 1.15 to 34.40, depending on the industrial sector. Welsch (2006)'s estimate for mean Armington elasticities for France ranges from negative 2.060 to

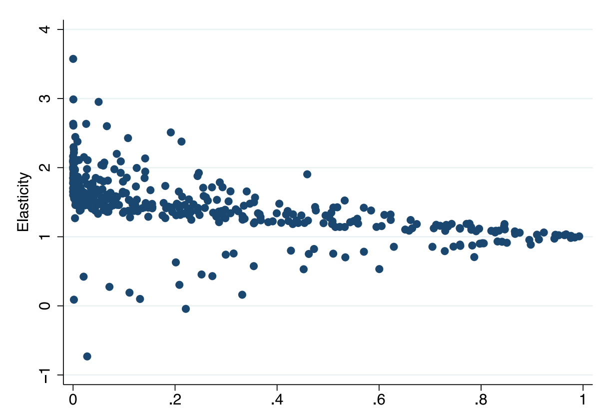

Fig. 1. CES elasticity vs significance (Japan).