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An evaluation of some popular investment strategies under stochastic interest rates

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Abstract

The payoff distribution pricing model (PDPM) of Dybvig [13] is a powerful tool for measuring the inefficiency of any investment strategy in a multiperiod setting. In this study, we extend the PDPM in three major ways. Firstly, we develop an operational formula for computing the inefficiency amount of a strategy. Secondly, we use six different investment horizons spanning from one month to five years to cater to short-term and long-term investors. Thirdly, and most importantly, we incorporate the stochastic nature of the short interest rate into the PDPM using two well-known interest rate models. Under such formulation, we investigate the inefficiency of three popular investment strategies. Our simulation results show that their inefficiency amounts increase considerably when the investment horizon lengthens and/or when the short interest rate is stochastic. In general, the stop-loss strategy performs better than the other two strategies in terms of inefficiency amount.

Keywords: Investment strategies; Inefficiency amount; Investment horizon; Interest rate models; Monte Carlo simulation

1. Introduction

In his seminal paper, Dybvig [13] developed the payoff distribution pricing model (PDPM) and used it as a performance yardstick to measure the inefficiency of investment strategies whose terminal payoffs are NOT in reverse order of the terminal state price densities (simply stated, the state price density is the state price divided by the objective probability). In a nutshell, taking risk and return together, the PDPM efficiency measures the difference (hereafter referred to as inefficiency amount) between the cost of a strategy producing a given distribution of terminal payoff and that of the corresponding efficient strategy producing the same distribution. The PDPM efficiency is different from the CAPM efficiency [22,28]. In the CAPM context, efficient portfolios are those frontier portfolios with expected returns strictly higher than that of the minimum variance portfolio. In the PDPM context, a portfolio is efficient provided its terminal payoffs are in reverse order of the terminal state price densities. Whereas the CAPM concerns optimization over a single period, the PDPM concerns optimization over multiple periods.

The PDPM is fully consistent with the generally accepted expected utility theory of Von Neumann and Morgenstern [34]. Nevertheless, the essence of the PDPM that inefficiency will result if terminal payoffs of a strategy are not in

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reverse order of the terminal state price densities calls for some explanations. Suppose there are m states at terminal time T. Let the m states be ranked in order of payoff, where state 1 has the lowest payoff and state m has the highest payoff. Since investors in the market as a whole are assumed risk-averse, dollar payoffs are more valuable in lower states than in higher states. Hence, they are willing to pay more for Arrow–Debreu securities [3,11] or pure securities in lower states than in higher states. Accordingly, if all the states are equally probable, state prices or state price densities are larger in lower states than in higher states. Hence, a strategy whose terminal payoffs are not in reverse order of the terminal state price densities will result in certain amount of inefficiency.

The PDPM is a powerful technology for measuring the inefficiency of any investment strategy in a multiperiod setting. According to the PDPM, many popular strategies (e.g., stop-loss, lock-in, and random market timing), designed supposedly to achieve certain investment objectives, are inefficient. In fact, Dybvig [13] found that the inefficiency amounts of these strategies are quite substantial. However, Dybvig embodied his PDPM in a simple formulation involving only short-term strategies with cash and stock. Specifically, to compute the inefficiency amounts of these strategies with a one-year investment horizon, he used the geometric Brownian motion for the stock price and a constant eight percent for the short interest rate. As always, the short interest rate (or simply the short rate) changes constantly and unpredictably. Hence, his simple formulation – with the short rate set at eight percent over the entire investment horizon – gives the impression that the practical applicability of the PDPM is limited.

In this study, we extend the PDPM in three major ways. Firstly, we develop an operational formula for computing the inefficiency amount of any strategy. Secondly, unlike Dybvig who used merely a one-year investment horizon, we use six different investment horizons spanning from one month to five years to cater to short-term and long-term investors because inefficiency amount does not vary proportionally with the length of the investment horizon. Thirdly, and most importantly, we formulate our PDPM in a more general and realistic framework. Specifically, we employ two commonly used interest rate models to characterize the stochastic nature of the short rate. One is the Cox–Ingersoll–Ross (CIR) model [8–10] and the other is the Vasicek model [33]. An important reason for using them together in this study is that the two models involve the same parameters (see Eqs. (6) and (7) in Section 3). As such, the inefficiency amounts obtained under the two models are readily comparable.

For implementation, we arrange three different cases for the evolution of the short rate over the six investment horizons: the first is that we assume constant short rate (as used by Dybvig), the second is that we assume the CIR model for the short rate, and the third is that we assume the Vasicek model for the short rate. That said, this study sets out to investigate the inefficiency under the three cases of the following three popular investment strategies: stop-loss, lock-in, and random market timing.

The rest of the paper proceeds as follows. In Section 2, we first give a simple PDPM example and then develop an operational formula for computing the amount of inefficiency of a strategy. In Section 3, we derive a formula for the terminal state price density for a strategy based on the geometric Brownian motion for the stock price and either the CIR model or the Vasicek model for the short rate. Section 4 estimates the relevant parameters using the maximum likelihood method. Section 5 gives a detailed description of the three strategies. In Section 6, we lay out our simulation design and, based the estimated parameter values, compute through Monte Carlo simulation the inefficiency amounts of the three strategies. Section 7 concludes this research.

2. An operational formula for inefficiency amount

In this section, we first give a simple example to bring out the idea of the PDPM and then develop an operational formula for the inefficiency amount of a strategy. To begin, note that the three assumptions of the PDPM are: (1) the preferences of investors depend only on the probability distribution of terminal payoff; (2) investors are risk-averse and prefer more payoff to less; and (3) the market is perfect and complete over finitely many equally probable terminal states. In the following, we will use PDPM market environment to describe a market that satisfies these three assumptions.

2.1. A simple PDPM example

Consider a strategy A with the following setup. Let the market be one-period with interest rate = 0. Let there be two equally probable states 1 and 2 at time 1 with state 1 price = $s_{01}(1) = \frac{2}{3}$ and state 2 price = $s_{01}(2) = \frac{1}{3}$. Strategy A is that at time 1 its payoff in state 1 is $V_1^A(1) = \frac{6}{5}$ and its payoff in state 2 is $V_1^A(2) = \frac{3}{5}$. That is, the terminal payoff