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Stochastic economic model predictive control for Markovian switching systems ⋆

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Abstract: The optimization of process economics within the model predictive control (MPC) formulation has given rise to a new control paradigm known as economic MPC (EMPC). Several authors have discussed the closed-loop properties of EMPC-controlled deterministic systems, however, little have uncertain systems been studied. In this paper we propose EMPC formulations for nonlinear Markovian switching systems which guarantee recursive feasibility, asymptotic performance bounds and constrained mean square (MS) stability.

Keywords: Stochastic control; Economic model predictive control; Markovian switching systems; Stochastic dissipativity.

1. INTRODUCTION

1.1 Background and motivation

Recently, a new approach to model predictive control (MPC) termed *economic model predictive control* (EMPC) has gained a lot of attention. Rather than minimizing a deviation from a prescribed (optimal/best) set-point or a tracking reference, the main objective in EMPC is to optimize a given economic cost functional (Angeli et al., 2012). Often, in engineering practice, the main objective is to devise control algorithms which asymptotically guarantee an economic operation of the controlled plant.

Already, a considerable body of theoretical results has been reported in the literature characterizing the asymptotic performance of EMPC. Perhaps *dissipativity* is the most salient notion in the pertinent literature which is shown to be a sufficient condition for proving optimal operation at a steady state and stability of EMPC formulations (Angeli et al., 2012). The same authors show that economic MPC has no worse an asymptotic average performance than the best admissible steady state operation (Müller et al., 2013).

The introduction of a, possibly non-quadratic and nonconvex, economic cost into the MPC framework disqualifies the standard stability analysis used in the MPC literature. Angeli et al. (2012) propose the use of a simple terminal constraint to guarantee stability of EMPC-controlled systems which is generalized by Amrit et al. (2011) using terminal set constraints. Fagiano and Teel (2013) use a generalized terminal state equality constraint where the target terminal state is left as a free variable to be optimized which increases the feasibility region of EMPC. This concept was further generalized to include a terminal region constraint (Müller et al., 2014). It was further shown that EMPC can achieve near-optimal operation without terminal constraints and costs for a sufficiently large prediction horizon (Grüne, 2013). Similar results exist for a system that is best operated at a periodic regime (Zanon et al., 2013). It is worth noting that this wealth of results concerns only deterministic systems.

In spite of the noticeable interest for the idea of EMPC there are very few theoretical results accounting for uncertainty, which is often relevant in a real-world operation. Bø and Johansen (2014) propose a scenario-based EMPC formulation for fault-tolerant constrained regulation and a similar approach is pursued by Lucia et al. (2014a). Lucia et al. (2014b) present a multi-stage scenario-based nonlinear MPC control strategy validated on a benchmark example, but no performance guarantees or stability analysis is provided. An interesting theoretical treatment is given by Bayer et al. (2014) where a tube-based EMPC formulation is proposed for constrained systems with bounded additive disturbances. Very recently Bayer et al. (2016) proposed a robust economic MPC formulation for linear systems with bounded additive uncertainty with known probability distribution.

1.2 Contributions

In this paper we endeavor to cover the theoretical gap in EMPC for an important class of stochastic systems — the Markovian switching systems. We first study the properties of an MPC formulation for Markovian switching systems where optimal steady states are mode-dependent. We propose an MPC scheme which is recursively feasible and satisfies an asymptotic performance bound. Assuming

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that there is a common optimal steady state, we show that the MPC-controlled system is mean-square (MS) stable when a stochastic dissipativity condition is satisfied. We then formulate a variant of the MPC problem using mode-dependent terminal constraints and provide mean-square stability conditions and performance bounds. We then provide guidelines for the design of mean-square stabilizing predictive controllers for nonlinear systems imposing weak conditions on the system dynamics and the EMPC stage cost.

1.3 Notation and mathematical preliminaries

Let $\mathbb{R}$ and $\mathbb{R}_+$, $\mathbb{R}^n$, $\mathbb{R}^{n\times n}$ denote the sets of real numbers, nonnegative reals, n-dimensional real vectors and n-by-m matrices. Let $\mathcal{B}_\delta$ be the ball of $\mathbb{R}$ of radius δ, that is $\mathcal{B}_\delta := \{x : \|x\| < \delta\}$. A function $f : \mathbb{R}^n \to \mathbb{R}$ is called *lower semicontinuous* if its epigraph, that is the set $\operatorname{epi} f = \{(x, \alpha) \in \mathbb{R}^{n+1} : f(x) \leq \alpha\}$, is closed. We say that $f : \mathbb{R}^n \to \mathbb{R}$ is *level-bounded* if its level sets, $\operatorname{lev}_\alpha f = \{x : f(x) \leq \alpha\}$, are bounded. We say that $f : \mathbb{R}^n \times \mathbb{R}^m \ni (x, u) \mapsto f(x, u) \in \mathbb{R}$ is *level-bounded in* u *locally uniformly in* x if for every $\bar{x}$ there is a neighborhood of $\bar{x}$, $V_{\bar{x}} \subseteq \mathbb{R}^n$, so that $\{(x, u) : x \in V_{\bar{x}}, f(x, u) \leq \alpha\}$ is bounded. A function $f : \mathbb{R}^n \to \mathbb{R}^m$ is called β*-smooth* if it is differentiable with β-Lipschitz gradient, that is $\|\nabla f(y) - \nabla f(x)\| \leq \beta\|y - x\|$ for all $x, y \in \mathbb{R}^n$; then, we have that $\|f(y) - f(x) - \nabla f(x)(y - x)\| \leq \frac{\beta}{2}\|y - x\|^2$. We say that a function $f : \mathbb{R}^n \to \mathbb{R}$ is positive definite around x_0 if $f(x_0) = 0$ and $f(x) > 0$ for $x \neq x_0$. $A \succcurlyeq 0$ denotes that A is a positive semidefinite matrix and $A \succ 0$ means that A is positive definite. We denote the transpose of a matrix A by $A^\top$.

2. STOCHASTIC ECONOMIC MODEL PREDICTIVE CONTROL

2.1 System dynamics

Consider the following Markovian switching system

$$x_{k+1} = f(x_k, u_k, \theta_k), \tag{1}$$

driven by the random parameter θ_k which is a time-homogeneous irreducible and aperiodic Markovian process with values in a finite set $\mathcal{N} = \{1, \ldots, \nu\}$ with transition matrix $P = (p_{ij}) \in \mathbb{R}^{\nu\times\nu}$ and *initial distribution* $v = (v_1, \ldots, v_\nu)$ (Costa et al., 2005). We assume that at time k we measure the full state x_k and the value of θ_k. Markov jump linear systems (MJLS) with additive disturbances are a special case of (1) with $f(x, u, \theta) = A_\theta x + B_\theta u + w_\theta$.

Let $\Omega := \prod_{k\in\mathbb{N}}(\mathbb{R}^n \times \mathbb{R}^m \times \mathcal{N})$ and $\mathfrak{F}_k$ be the minimal σ-algebra over the Borel-measurable rectangles of Ω with k-dimensional base and $\mathfrak{F}$ be the minimal σ-algebra over all Borel-measurable rectangles. Define the filtered probability space $(\Omega, \mathfrak{F}, \{\mathfrak{F}_k\}_{k\in\mathbb{N}}, \mathrm{P})$ where P is the unique product probability measure according to (Ash, 1972, Th. 2.7.2) with $\mathrm{P}(\theta_0 = i_0, \theta_1 = i_1, \ldots, \theta_k = i_k) = v_{i_0} p_{i_0 i_1} \cdots p_{i_{k-1} i_k}$ for any $i_0, i_1, \ldots, i_k \in \mathcal{N}$ and $k \in \mathbb{N}$, where θ_k is an $\mathfrak{F}_k$-adapted random variable from Ω to $\mathcal{N}$. We will use the notation $u \lhd \mathfrak{F}_k$ to denote that the random variable u is $\mathfrak{F}_k$-measurable.

Let $\mathbb{E}[\cdot]$ denote the expectation of a random variable with respect to P and $\mathbb{E}[\cdot|\mathfrak{F}_k]$ the conditional expectation. It can be shown (Tejada et al., 2010) that the augmented state (x_k, θ_k) contains all the probabilistic information relevant to the evolution of the Markovian switching system for all time instants $t > k$.

Definition 1. (Cover and bet node). For every node $i \in \mathcal{N}$, the *cover* of i is the set $\mathcal{C}(i) = \{j \in \mathcal{N} \mid p_{ij} > 0\}$. The *bet node* of an $i \in \mathcal{N}$ is a node $\operatorname{bet}(i) \in \mathcal{C}(i)$ with $p_{i\operatorname{bet}(i)} \geq p_{ij}$ for all $j \in \mathcal{C}(i)$.

A bet of a mode $\theta_k = i$ is one of the most likely successor modes θ_{k+1}.

System (1) is subject to the following joint state-input constraints

$$(x_k, u_k) \in Y_{\theta_k}. \tag{2}$$

Let $\ell : \mathbb{R}^n \times \mathbb{R}^m \times \mathcal{N} \to \mathbb{R}$ be a mode-dependent cost function.

Assumption 1. (Well-posedness). For each $\theta \in \mathcal{N}$, $\ell(\cdot, \cdot, \theta)$ are nonnegative, lower semicontinuous and level-bounded in u locally uniformly in x, $f(\cdot, \cdot, \theta)$ are continuous and sets Y_θ are nonempty and compact. The random process $\{\theta_k\}_k$ is an irreducible and aperiodic Markov chain.

Definition 2. (Optimal steady states). Given a stage cost function $\ell : \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{N} \to \mathbb{R}$ which satisfies Assumption 1, a pair (x_s^θ, u_s^θ) is called an *optimal steady state* of (1) subject to (2) with respect to ℓ if it is a minimizer of the problem

$$\ell_s(\theta) := \min_{x,u} \{\ell(x, u, \theta) | f(x, u, \theta) = x, (x, u) \in Y_\theta\}$$

For reasons that will be better elucidated in the next section, we need to draw the following controllability assumption essentially requiring that if $x_k = x_s^i$ and $\theta_k = j$ then there is a control action $\bar{u}_s^{i,j}$ so that at time $k+1$ the state is steered to $x_{k+1} = x_s^{\operatorname{bet}(j)}$.

Assumption 3. (Controllability). In addition to Assumption 1, for all $i, j \in \mathcal{N}$ there is a control law $\bar{u}_s : \mathbb{R}^n \times \mathcal{N} \to \mathbb{R}^m$ with $\bar{u}_s(x_s^i, j) = \bar{u}_s^{i,j}$ so that $(x_s^i, \bar{u}_s^{i,j}) \in Y_j$ and $f(x_s^i, \bar{u}_s^{i,j}, j) = x_s^{\operatorname{bet}(j)}$.

2.2 Model predictive control

In this section we shall present a model predictive control framework for constrained Markovian switching systems with mode-dependent optimal steady-state points.

Let $u_k \lhd \mathfrak{F}_k$ and $\mathbf{u}_N = (u_0, \ldots, u_{N-1})$, and define

$$V_N(x_0, \theta_0, \mathbf{u}_N) = \mathbb{E}\left[V_f(x_N, \theta_N) + \sum_{j=0}^{N-1} \ell(x_j, u_j, \theta_j) \middle| \mathfrak{F}_0\right].$$

Here, we take $V_f = 0$ and let the state sequence satisfy (1).

We introduce the following stochastic economic model predictive control problem

$$\mathbb{P}(x, \theta) : V_N^\star(x, \theta) = \inf_{\mathbf{u}_N} V_N(x, \theta, \mathbf{u}_N), \tag{3a}$$

and for $k = 0, \ldots, N-1$, subject to

$$x_{k+1} = f(x_k, u_k, \theta_k) \tag{3b}$$

$$(x_k, u_k) \in Y_{\theta_k} \tag{3c}$$

$$(x_0, \theta_0) = (x, \theta) \tag{3d}$$

$$x_N = x_s^{\operatorname{bet}(\theta_{N-1})} \tag{3e}$$

$$u_k \lhd \mathfrak{F}_k. \tag{3f}$$