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# Stochastic economic model predictive Prochastic control for Markovian switching systems<sup>\*</sup>  $Stochastic economic model predictive\n\n... \n\n% of the M_N is a set of the data.$

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 $\mathcal{A}$  ,  $\mathcal{A}$  is the optimization in Eq. (

Abstract: The optimization of process economics within the model predictive control (MPC) Several authors have discussed the closed-loop properties of EMPC-controlled deterministic systems, however, little have uncertain systems been studied. In this paper we propose EMPC systems, however, here have uncertain systems been studied. In this paper we propose EMI C<br>formulations for nonlinear Markovian switching systems which guarantee recursive feasibility, normalations for noninear markovian switching systems which guarantee recursive reasionity, asymptotic performance bounds and constrained mean square (MS) stability. for  $\mathbf{f}$  and  $\mathbf{f}$  are cursive feasibility, systems which guarantee recursive feasibility,  $\mathbf{f}$  $\epsilon$ . The optimization of process economics within the model predictive control (MPC).<br>formulation has given rise to a new control paradigm known as economic MPC (EMPC). asymptotic performance bounds and constrained mean square (MS) stability. formulation has given rise to a new control paradigm known as economic MPC (EMPC). Several authors have discussed the closed-loop properties of  $EMPC$ -controlled deterministic systems, however, little have uncertain systems been studied. In this paper we propose  $EMPC$ formulations for nonlinear Markovian switching systems which guarantee recursive feasibility,

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Keywords: Stochastic control; Economic model predictive control; Markovian switching systems; Stochastic dissipativity. systems; Stochastic control; Economic model predictive control; Markovian switching<br>Keywords: Stochastic control; Economic model predictive control; Markovian switching target terminal state is left as a free variable to be optirey words. Stochastic control, Eq. Keywords: Stochastic control; Economic model predictive control; Markovian switching systems; Stochastic dissipativity.

#### 1. INTRODUCTION 1. International control of the con 1. INTRODUCTION 1. INTRODUCTION

#### 1.1 Background and motivation  $\mathcal{L}$  $1.1$  Background and motivation 1.1 Background and motivation

Recently, a new approach to model predictive control developed a lot of a lot of about the model predictive control (EMPC) termed economic model predictive control (EMPC)  $(mC)$  terms *economic model predictive control*  $(EMC)$  has gained a lot of attention. Rather than minimizing a has gained a fot of attention. Rather than minimizing a deviation from a prescribed (optimal/best) set-point or deviation from a prescribed (optimal/best) set-point of<br>a tracking reference, the main objective in EMPC is to optimize a given economic cost functional (Angeli et al., Springer et al., 2012). Often, in engineering practice, the main objective zorz). Onen, in engineering practice, the main objective<br>is to devise control algorithms which asymptotically guarantee an economic operation of the controlled plant. (MPC) termed economic model predictive control (EMPC) Recently, a new approach to model predictive control antee an economic operation of the controlled plant. Recently, a new approach to model predictive control (MPC) termed economic model predictive control (EMPC) has gained a lot of attention. Rather than minimizing a deviation from a prescribed (optimal/best) set-point or a tracking reference, the main objective in EMPC is to optimize a given economic cost functional (Angeli et al., 2012). Often, in engineering practice, the main objective is to devise control algorithms which asymptotically guarantee an economic operation of the controlled plant.

Already, a considerable body of theoretical results has Alleaux, a considerable body of theoretical results has been reported in the interature characterizing the asymptotic performance of EMPC. Perhaps *dissipativity* is the front performance of EMPC. Temaps *assiguating* is the most salient notion in the pertinent literature which is must same a sufficient condition for proving optimal shown to be a sufficient condition for proving optimal shown to be a sumclem condition for proving optimals<br>operation at a steady state and stability of EMPC foroperation at a steady state and stability of EMTC formulations (Angeli et al., 2012). The same authors show  $\mu$ uller et al., 2012). The same authors show that economic MPC has no worse an asymptotic average  $\frac{1}{2}$  and  $\frac{1}{2}$  contoint  $\frac{1}{2}$  or al.  $\frac{1}{2}$  of  $\frac{1}{2}$  or  $\$  $\frac{1}{1}$ tion (Müller et al., 2013). been reported in the literature characterizing the asymp-Already, a considerable body of theoretical results has periormance than the bes<br>tion (Müller et al., 2012). Already, a considerable body of theoretical results has been reported in the literature characterizing the asymptotic performance of EMPC. Perhaps dissipativity is the most salient notion in the pertinent literature which is shown to be a sufficient condition for proving optimal operation at a steady state and stability of EMPC formulations (Angeli et al., 2012). The same authors show that economic MPC has no worse an asymptotic average performance than the best admissible steady state operation (Muller et al.,  $2013$ ).

The introduction of a, possibly non-quadratic and noncon-The introduction of a, possibly non-quadratic and noncon-<br>vex, economic cost into the MPC framework disqualifies the standard stability analysis used in the MPC literature. Angeli et al. (2012) propose the use of a simple terminal constraint to guarantee stability of EMPC-controlled sys $t_{\text{c}}$  tems which is generalized by Amrit et al. (2011) using tems which is generalized by Amrit et al. (2011) terminal set constraints. Fagiano and Teel (2011) using<br>terminal set constraints. Fagiano and Teel (2013) use a terminal set constraints. Fagiano and Teer (2013) use a generalized terminal state equality constraint where the vex, economic cost into the MPC framework disputes  $\mathcal{L}_{\mathcal{A}}$ The introduction of a, possibly non-quadratic and noncongeneralized terminal state equality constraint where the Angeli et al. (2012) propose the use of a simple terminal The introduction of a, possibly non-quadratic and nonconvex, economic cost into the MPC framework disqualifies the standard stability analysis used in the MPC literature. tems which is generalized by Amrit et al. (2011) using terminal set constraints. Fagiano and Teel (2013) use a generalized terminal state equality constraint where the target terminal state is left as a free variable to be optitaiget terminal state is left as a free variable to be opti-<br>mized which increases the feasibility region of EMPC. This encept was further generalized to include a terminal reconcept was further generalized to include a terminal region constraint (Müller et al., 2014). It was further shown that EMPC can achieve near-optimal operation without terminal constraints and costs for a sufficiently large prediction horizon (Grüne, 2013). Similar results exist for a et al., 2013). It is worth noting that is best operated at a periodic regime (Zanon system that is best operated at a periodic regime (zanon<br>et al., 2013). It is worth noting that this wealth of results concerns only deterministic systems. mized which increases the feasibility region of EMPC. This target terminal state is left as a free variable to be opticoncerns only deterministic systems. target terminal state is left as a free variable to be optimized which increases the feasibility region of EMPC. This concept was further generalized to include a terminal region constraint (Müller et al., 2014). It was further shown that EMPC can achieve near-optimal operation without terminal constraints and costs for a sufficiently large prediction horizon (Grune, 2015). Similar results exist for a system that is best operated at a periodic regime (Zanon et al., 2013). It is worth noting that this wealth of results concerns only deterministic systems.

target terminal state is left as a free variable to be opti-

In spite of the noticeable interest for the idea of EMPC there are very few theoretical results accounting for uncertainty, which is often relevant in a real-world operation.  $B\phi$  and Johansen (2014) propose a scenario-based EMPC by and Johansen (2014) propose a scenario-based EMI C<br>formulation for fault-tolerant constrained regulation and a of mulation for fault-colerant constrained regulation and a<br>similar approach is pursued by Lucia et al. (2014a). Lucia et al. (2014b) present a multi-stage scenario-based nonlinet al. (2014b) present a mutu-stage scenario-based nonm-<br>ear MPC control strategy validated on a benchmark exear MTC control strategy vandated on a benchmark example, but no performance guarantees or stability analysis ample, but no performance guarantees of stability analysis<br>is provided. An interesting theoretical treatment is given is provided. An interesting theoretical treatment is given<br>by Bayer et al. (2014) where a tube-based EMPC formuby Bayer et al. (2014) where a tube-based EMI C formulation is proposed for constrained systems with bounded additive disturbances. Very recently Bayer et al. (2016)<br>additive disturbances. Very recently Bayer et al. (2016) additive distributions. Very recently bayer et al. (2010)<br>proposed a robust economic MPC formulation for linear proposed a robust economic MTC formulation for mean<br>systems with bounded additive uncertainty with known probability distribution. there are very few theoretical results accounting for un-In spite of the noticeable interest for the idea of  $EMPC$ et al. (2014b) present a multi-stage scenario-based nonlin-similar approach is pursued by Lucia et al. (2014a). Lucia probability distribution.<br>Systems with distribution. In spite of the noticeable interest for the idea of EMPC there are very few theoretical results accounting for uncertainty, which is often relevant in a real-world operation. Bø and Johansen (2014) propose a scenario-based EMPC formulation for fault-tolerant constrained regulation and a ear MPC control strategy validated on a benchmark example, but no performance guarantees or stability analysis is provided. An interesting theoretical treatment is given by Bayer et al. (2014) where a tube-based EMPC formulation is proposed for constrained systems with bounded additive disturbances. Very recently Bayer et al. (2016) proposed a robust economic MPC formulation for linear systems with bounded additive uncertainty with known probability distribution.

#### 1.2 Contributions  $\cdots$  cover to accover 1.2 Contributions

In this paper we endeavor to cover the theoretical gap In this paper we enterated to cover the theoretical gap<br>in EMPC for an important class of stochastic systems. m EMI C for an Important class of stochastic systems.<br>— the Markovian switching systems. We first study the — the Markovian switching systems. We first study the properties of an MPC formulation for Markovian switching properties of an MPC formulation for Markovian switching<br>systems where optimal steady states are mode-dependent. systems where optimal steady states are mode-dependent.<br>We propose an MPC scheme which is recursively feasible and satisfies an asymptotic performance bound. Assuming and satisfies an asymptotic performance bound. Assuming In this paper we endeavor to cover the theoretical gap in EMPC for an important class of stochastic systems — the Markovian switching systems. We first study the properties of an MPC formulation for Markovian switching systems where optimal steady states are mode-dependent. We propose an MPC scheme which is recursively feasible and satisfies an asymptotic performance bound. Assuming

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Authorized Communication Council under the Council under the Council under the Council under the  $\star$  This work was supported by the EU-funded H2020 research This work was supported by the EU-funded H2020 research<br>project DISIRE, grant agreement No. 636834. The work of the third project DISIKE, grant agreement No. 030834. The work of the third<br>author was supported by the KU Leuven Research Council under BOF/STG-15-043. BOF/STG-15-043. **Company is to 316.** BOF/STG-15-043. author was supported by the KU Leuven Research Council under

that there is a common optimal steady state, we show that the MPC-controlled system is mean-square (MS) stable when a stochastic dissipativity condition is satisfied. We then formulate a variant of the MPC problem using modedependent terminal constraints and provide mean-square stability conditions and performance bounds. We then provide guidelines for the design of mean-square stabilizing predictive controllers for nonlinear systems imposing weak conditions on the system dynamics and the EMPC stage cost.

### 1.3 Notation and mathematical preliminaries

Let  $\mathbb{R}$  and  $\mathbb{R}_+$ ,  $\mathbb{R}^n$ ,  $\mathbb{R}^{n \times n}$  denote the sets of real numbers, nonnegative reals, n-dimensional real vectors and n-bym matrices. Let  $\mathcal{B}_{\delta}$  be the ball of R of radius  $\delta$ , that is  $\mathcal{B}_{\delta} := \{x : ||x|| < \delta\}$ . A function  $f : \mathbb{R}^{n} \to \mathbb{R}$  is called lower semicontinuous if its epigraph, that is the set epi  $f = \{(x, \alpha) \in \mathbb{R}^{n+1} : f(x) \leq \alpha\}$ , is closed. We say that  $f : \mathbb{R}^n \to \mathbb{R}$  is *level-bounded* if its level sets,  $lev_{\alpha} f = \{x : f(x) \leq \alpha\}$ , are bounded. We say that  $f: \mathbb{R}^n \times \mathbb{R}^m \ni (x, u) \mapsto f(x, u) \in \mathbb{R}$  is level-bounded in u locally uniformly in x if for every  $\bar{x}$  there is a neighborhood of  $\bar{x}, V_{\bar{x}} \subseteq \mathbb{R}^n$ , so that  $\{(x, u) : x \in V_{\bar{x}}, f(x, u) \leq \alpha\}$  is bounded. A function  $f : \mathbb{R}^n \to \mathbb{R}^m$  is called  $\beta$ -smooth if it is differentiable with  $\beta$ -Lipschitz gradient, that is  $\|\nabla f(y) - \nabla f(x)\| \leq \beta \|y - x\|$  for all  $x, y \in \mathbb{R}^n$ ; then, we have that  $|| f(y) - f(x) - \nabla f(x)(y-x) || \leq \frac{\beta}{2} ||y-x||^2$ . We say that a function  $f : \mathbb{R}^n \to \mathbb{R}$  is positive definite around  $x_0$  if  $f(x_0) = 0$  and  $f(x) > 0$  for  $x \neq x_0$ .  $A \ge 0$ denotes that A is a positive semidefinite matrix and  $A \succ 0$ means that A is positive definite. We denote the transpose of a matrix  $A$  by  $A^{\perp}$ .

# 2. STOCHASTIC ECONOMIC MODEL PREDICTIVE **CONTROL**

## 2.1 System dynamics

Consider the following Markovian switching system

$$
x_{k+1} = f(x_k, u_k, \theta_k), \tag{1}
$$

driven by the random parameter  $\theta_k$  which is a timehomogeneous irreducible and aperiodic Markovian process with values in a finite set  $\mathcal{N} = \{1, \ldots, \nu\}$  with transition matrix  $P = (p_{ij}) \in \mathbb{R}^{\nu \times \nu}$  and *initial distribution*  $v =$  $(v_1,\ldots,v_\nu)$  (Costa et al., 2005). We assume that at time k we measure the full state  $x_k$  and the value of  $\theta_k$ . Markov jump linear systems (MJLS) with additive disturbances are a special case of (1) with  $f(x, u, \theta) = A_{\theta}x + B_{\theta}u + w_{\theta}$ . Let  $\Omega := \prod_{k \in \mathbb{N}} (\mathbb{R}^n \times \mathbb{R}^m \times \mathcal{N})$  and  $\mathfrak{F}_k$  be the minimal σ-algebra over the Borel-measurable rectangles of Ω with k-dimensional base and  $\mathfrak F$  be the minimal  $\sigma$ -algebra over all Borel-measurable rectangles. Define the filtered probability space  $(\Omega, \mathfrak{F}, \{\mathfrak{F}_k\}_{k\in \mathbb{N}}, P)$  where P is the unique product probability measure according to (Ash, 1972, Th. 2.7.2) with  $P(\theta_0 = i_0, \theta_1 = i_1, \dots, \theta_k = i_k) = v_{i_0} p_{i_0 i_1} \cdots p_{i_{k-1} i_k}$ for any  $i_0, i_1, \ldots, i_k \in \mathcal{N}$  and  $k \in \mathbb{N}$ , where  $\theta_k$  is an  $\mathfrak{F}_k$ adapted random variable from  $\Omega$  to N. We will use the notation  $u \triangleleft \mathfrak{F}_k$  to denote that the random variable u is  $\mathfrak{F}_k$ -measurable.

Let  $\mathbb{E}[\cdot]$  denote the expectation of a random variable with respect to P and  $\mathbb{E}[\cdot|\mathfrak{F}_k]$  the conditional expectation. It can

be shown (Tejada et al., 2010) that the augmented state  $(x_k, \theta_k)$  contains all the probabilistic information relevant to the evolution of the Markovian switching system for all time instants  $t > k$ .

Definition 1. (Cover and bet node). For every node  $i \in$ N, the cover of i is the set  $C(i) = \{j \in \mathcal{N} \mid p_{ij} > 0\}.$ The bet node of an  $i \in \mathcal{N}$  is a node bet $(i) \in \mathcal{C}(i)$  with  $p_{i\text{bet}(i)} \geq p_{ij}$  for all  $j \in \mathcal{C}(i)$ .

A bet of a mode  $\theta_k = i$  is one of the most likely successor modes  $\theta_{k+1}$ .

System (1) is subject to the following joint state-input constraints

$$
(x_k, u_k) \in Y_{\theta_k}.\tag{2}
$$

Let  $\ell : \mathbb{R}^n \times \mathbb{R}^m \times \mathcal{N} \to \mathbb{R}$  be a mode-dependent cost function.

Assumption 1. (Well-posedness). For each  $\theta \in \mathcal{N}$ ,  $\ell(\cdot, \cdot, \theta)$ are nonnegative, lower semicontinuous and level-bounded in u locally uniformly in x,  $f(\cdot, \cdot, \theta)$  are continuous and sets  $Y_{\theta}$  are nonempty and compact. The random process  ${\lbrace \theta_k \rbrace_k}$  is an irreducible and aperiodic Markov chain.

Definition 2. (Optimal steady states). Given a stage cost function  $\ell : \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{N} \to \mathbb{R}$  which satisfies Assumption 1, a pair  $(x_s^{\theta}, u_s^{\theta})$  is called an *optimal steady state* of (1) subject to (2) with respect to  $\ell$  if it is a minimizer of the problem

$$
\ell_s(\theta) := \min_{x,u} \left\{ \ell(x,u,\theta) | f(x,u,\theta) = x, (x,u) \in Y_{\theta} \right\}
$$

For reasons that will be better elucidated in the next section, we need to draw the following controllability assumption essentially requiring that if  $x_k = x_s^i$  and  $\theta_k = j$ then there is a control action  $\bar{u}_s^{i,j}$  so that at time  $k+1$  the state is steered to  $x_{k+1} = x_s^{\text{bet}(j)}$ .

Assumption 3. (Controllability). In addition to Assumption 1, for all  $i, j \in \mathcal{N}$  there is a control law  $\bar{u}_s : \mathbb{R}^n \times$  $\mathcal{N} \to \mathbb{R}^m$  with  $\bar{u}_s(x_s^i, j) = \bar{u}_s^{i,j}$  so that  $(x_s^i, \bar{u}_s^{i,j}) \in Y_j$  and  $f(x_s^i, \bar{u}_s^{i,j}, j) = x_s^{\text{bet}(j)}.$ 

#### 2.2 Model predictive control

In this section we shall present a model predictive control framework for constrained Markovian switching systems with mode-dependent optimal steady-state points.

Let 
$$
u_k \triangleleft \mathfrak{F}_k
$$
 and  $\mathbf{u}_N = (u_0, \dots, u_{N-1})$ , and define  
\n
$$
V_N(x_0, \theta_0, \mathbf{u}_N) = \mathbb{E}\left[V_f(x_N, \theta_N) + \sum_{j=0}^{N-1} \ell(x_j, u_j, \theta_j) \Big| \mathfrak{F}_0\right].
$$

Here, we take  $V_f = 0$  and let the state sequence satisfy (1).

We introduce the following stochastic economic model predictive control problem

$$
\mathbb{P}(x,\theta): V_N^{\star}(x,\theta) = \inf_{\mathbf{u}_N} V_N(x,\theta,\mathbf{u}_N), \tag{3a}
$$

and for  $k = 0, \ldots, N-1$ , subject to

$$
x_{k+1} = f(x_k, u_k, \theta_k)
$$
\n(3b)

$$
(x_k, u_k) \in Y_{\theta_k} \tag{3c}
$$

$$
(x_0, \theta_0) = (x, \theta) \tag{3d}
$$

$$
x_N = x_s^{\text{bet}(\theta_{N-1})}
$$
 (3e)

$$
u_k \lhd \mathfrak{F}_k. \tag{3f}
$$

# ِ متن کامل مقا<mark>ل</mark>ه

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